Fluctuation-dissipation theorem and flux noise in overdamped Josephson-junction arrays

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The form of the fluctuation-dissipation theorem for a resistively shunted Josephson-junction array is derived with the help of the method which explicitly takes into account the screening effects. This result is used to express the flux noise power spectrum in terms of the frequency-dependent sheet impedance of the array. The relation between the noise amplitude and the parameters of the detection coil is analyzed for the simplest case of a single-loop coil, as well as the frequency dependence of the noise spectrum in different regimes.

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I. INTRODUCTION

The two basic experimental methods used for contactless investigation of the finite frequency properties of two-dimensional superconducting systems (such as thin films,[1–5] Josephson junction arrays,[6–9] and wire networks[10,11]) are the two-coil mutual-inductance technique[1–4,9–11] and the flux noise power spectrum analysis.[3–8] The first of them is based on the measurement of the voltage induced in the detection coil by the currents flowing in the sample under the action of the ac electric field produced by the current in the other (driving) coil. For the given geometry of the coils the measured signal can be used to extract[10] the complex frequency dependent sheet impedance $Z_\square(\omega)$ of the sample on the assumption that for wavelengths larger than the characteristic dimensions of the detection coil $Z_\square(\omega)$ is not wavelength dependent.

In the case of the flux noise spectrum analysis, the approaches to interpretation of the experimental data are much more varied. The theoretical predictions of the flux noise spectrum used for comparison with experimental data are found by relating it with[7,12] or (no less often) by replacing it by[3,13,14] a correlation function describing the vortex distribution and, naturally, they turn out to be dependent on the particular choice of assumptions concerning the form of this distribution. Numerical simulations also demonstrate a clear tendency towards studying the vortex number noise[13,15,16] rather then the flux noise. The only attempt to achieve a description of the flux noise power spectrum in terms of the sheet impedance of the sample taking into account the actual geometry of the detection coil has been undertaken by Kim and Minnhagen.[17] However, this calculation is also based on expressing all quantities in terms of the vortex gas correlation functions and, therefore, can be expected to be applicable only in a limited range of parameters.

In the present paper, we argue that in the case of a resistively shunted Josephson-junction array the general expression for the flux noise spectrum can be found without artificial decomposition of all fluctuations into the vortex part (which is usually assumed to be responsible for the flux noise) and the remaining so-called ‘‘spin-wave’’ part (which is traditionally neglected). Although in semiphenomenological treatment[12–14,18] of two-dimensional superconductors such decomposition seems to be inevitable, the case of an overdamped Josephson-junction array allows for the application of a more universal approach to the calculation of the flux noise power spectrum. It is based on the direct relation (discussed in Sec. II) of the flux noise with current fluctuations, which, on the other hand, can be expressed in terms of the complex frequency-dependent sheet impedance with the help of the fluctuation-dissipation theorem.

The additional advantage of such an approach is that it allows one to include into consideration in a systematic way the mutual influence between magnetic-field fluctuations and current fluctuations (the screening effects), which insofar has been neglected in the theoretical works[12–17] devoted to the flux noise spectrum analysis. The form of the Hamiltonian, which should be used for the description of a resistively shunted array in the presence of self-induced magnetic fields, is discussed in Sec. III, and the corresponding dynamic equations in Sec. IV.

The explicit form of the fluctuation-dissipation theorem for resistively shunted Josephson-junction array is derived in Sec. V. It shows that the current correlations in the array are determined by the response of the current to the external electric field and not directly by the sheet impedance of the array (which is defined as a response to the *total* electric field). The nature of the expression for the currents correlation function, related with the peculiarities of the two-dimensional geometry, allows one to expect the same expression to be applicable for arbitrary two-dimensional systems in which capacitive effects can be neglected.

Our main result, the relation between the flux noise power spectrum and the frequency-dependent sheet impedance of a two-dimensional superconductor, is presented in Sec. VI, which includes also the discussion of the noise spectrum frequency dependence and its relation with the parameters of the detection coil, as well as a comparison of our results with those of other authors.

II. FLUX NOISE AND CURRENT CORRELATIONS

In a flux noise experiment, one measures and analyzes the time dependence of a voltage created in a detection coil by fluctuations of currents in some conducting (or superconducting) object. This voltage is determined by the time derivative of the magnetic flux penetrating the coil, and the value of the flux can be expressed in terms of the current density distribution $\mathbf{j}(\mathbf{r})[\mathbf{r}=(x_1,x_2,x_3)]$ inside the object with the help of the Biot-Savart’s law, which in the Coulomb gauge ($\mathrm{div}\mathbf{A}=0$) can be written as

$$\Delta\mathbf{A}(\mathbf{r})=-\mu_0\mathbf{j}^t(\mathbf{r}), \tag{1}$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential defining the distribution of the magnetic field (magnetic induction) $\mathbf{B}(\mathbf{r})=\mathrm{rot}\,\mathbf{A}(\mathbf{r})$ created by $\mathbf{j}(\mathbf{r})$,

$$\Delta\equiv\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2} \tag{2}$$

is the three-dimensional Laplacian and

$$\mathbf{j}^t(\mathbf{r})\equiv\mathbf{j}(\mathbf{r})-\Delta^{-1}\mathrm{grad}\,\mathrm{div}\,\mathbf{j}(\mathbf{r}) \tag{3}$$

is the transverse part of $\mathbf{j}(\mathbf{r})$. Magnetic fields produced by the longitudinal part of $\mathbf{j}(\mathbf{r})$ cancel each other.

In the case of a system which can be considered as effectively two-dimensional and situated (for simplicity) in the plane $x_3=0$, the three-dimensional current density $\mathbf{j}(\mathbf{r})$ is reduced to

$$\mathbf{j}(\mathbf{r})=\mathbf{i}(\mathbf{x})\,\delta(x_3), \tag{4}$$

where $\mathbf{x}\equiv x_\alpha$ ($\alpha=1,2$) is the two-dimensional vector defining the position of a point in the plane $x_3=0$ and $\mathbf{i}\equiv i_\alpha$ is the two-dimensional vector describing the two-dimensional current density.

Substitution of Eq. (4) into Eq. (1) allows us then to find that

$$\mathbf{A}(\mathbf{r})=\mu_0\int\frac{d^2\mathbf{q}}{(2\pi)^2}\int\frac{dq_3}{2\pi}\,\frac{\exp\,i(\mathbf{qx}+q_3x_3)}{q^2+q_3^2}\mathbf{i}^t(\mathbf{q})$$
$$=\frac{\mu_0}{2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,\frac{\exp(i\mathbf{qx}-q|x_3|)}{q}\mathbf{i}^t(\mathbf{q}), \tag{5}$$

where

$$\mathbf{i}(\mathbf{q})=\int d^2\mathbf{x}\exp(-i\mathbf{qx})\mathbf{i}(\mathbf{x}) \tag{6}$$

is the (two-dimensional) Fourier transform of $\mathbf{i}(\mathbf{x})$, $q=|\mathbf{q}|$ and

$$\mathbf{i}^t(\mathbf{q})\equiv\mathbf{i}(\mathbf{q})-\hat{\mathbf{q}}[\hat{\mathbf{q}}\mathbf{i}(\mathbf{q})] \tag{7}$$

is the transverse part of $\mathbf{i}(\mathbf{q})$, $\hat{\mathbf{q}}\equiv\mathbf{q}/q$ being the unit vector parallel to $\mathbf{q}$.

In the simplest case, a coil can be approximated by a closed circular ring. Integration of $\mathbf{A}(\mathbf{r})$ over the perimeter of the ring $x_1^2+x_2^2=r^2$ situated at the distance h from the plane $x_3=0$ gives

$$\Phi=\oint d\mathbf{r}\,\mathbf{A}(\mathbf{r})=\frac{\mu_0}{2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}F(\mathbf{q})i^t(\mathbf{q}), \tag{8}$$

where

$$i^t(\mathbf{q})=\sum_{\alpha,\beta}\epsilon_{\alpha\beta}\hat{q}_\alpha i_\beta(\mathbf{q}) \tag{9}$$

is the amplitude of $\mathbf{i}^t(\mathbf{q})$, $\epsilon_{\alpha\beta}$ is the unit antisymmetric tensor,

$$F(\mathbf{q})=\frac{2\pi rJ_1(rq)}{q}\exp(-hq) \tag{10}$$

is the geometrical factor depending on the parameters of the coil, and $J_1(z)$ is the first-order Bessel function. In the case when the coil can be considered as consisting of N turns separated by the distance b from each other, the expression for $F(\mathbf{q})$ should also include the additional factor obtained by the summation of contributions from different turns[10]

$$F(\mathbf{q})=\frac{2\pi rJ_1(rq)}{q}\exp(-hq)\frac{1-\exp(-Nbq)}{1-\exp(-bq)}. \tag{11}$$

The power spectrum of the flux noise is given by the flux-flux correlation function

$$S(\omega)=\int dt\langle\Phi(t_0+t)\Phi(t_0)\rangle\exp(i\omega t) \tag{12}$$

and with the help of Eq. (8) can be expressed in terms of the current density correlation function

$$\langle\mathbf{i}^t\mathbf{i}^t\rangle_{\mathbf{q}\omega}\equiv\int d^2\mathbf{x}\int dt[\exp(-i\mathbf{qx}+i\omega t)$$
$$\times\langle\mathbf{i}^t(\mathbf{x_0}+\mathbf{x},t_0+t)\mathbf{i}^t(\mathbf{x_0},t_0)\rangle] \tag{13}$$

as

$$S(\omega)=\frac{\mu_0^2}{4}\int\frac{d^2\mathbf{q}}{(2\pi)^2}F^2(\mathbf{q})\langle\mathbf{i}^t\mathbf{i}^t\rangle_{\mathbf{q}\omega}. \tag{14}$$

III. HAMILTONIAN OF JOSEPHSON-JUNCTION ARRAY

When a self-induced magnetic field is taken into account, a square Josephson-junction array can be described by the Hamiltonian[19,20]

$$H=-J\sum_{\mathbf{n},\alpha}\cos(\nabla_\alpha\varphi_\mathbf{n}-A_{\mathbf{n}\alpha})$$
$$+\frac{1}{2}\sum_{\mathbf{n},\mathbf{k}}(\nabla\times A)_\mathbf{n}M^{-1}_{\mathbf{nk}}(\nabla\times A)_\mathbf{k}, \tag{15}$$

where $\varphi_\mathbf{n}$ is the phase of the order parameter on the $\mathbf{n}$th superconducting island, $\mathbf{n}\equiv(n_1,n_2)$ with n_1 and n_2 integers is the number of the island, the variables $A_{\mathbf{n}\alpha}$ (defined on the bonds of the lattice) are determined by the integral of the vector potential $\mathbf{A}(\mathbf{r})$ over the line connecting the geometrical centers of two neighboring islands

$$A_{\mathbf{n}\alpha}=\frac{2e}{\hbar}\int_{a\mathbf{n}}^{a(\mathbf{n}+\mathbf{e}_\alpha)}d\mathbf{r}\,\mathbf{A}(\mathbf{r}), \tag{16}$$

a is period of the lattice and

$$\mathbf{e}_\alpha=\begin{cases}(1,0) & \text{for}\quad\alpha=1\\(0,1) & \text{for}\quad\alpha=2\end{cases} \tag{17}$$

are the two unit vectors.

The first term in Eq. (15) describes the Josephson energy of the junctions in the array. The coupling constant J entering this term is determined by the critical current I_c of a single junction:

$$J=\frac{\hbar}{2e}I_c, \tag{18}$$

which is assumed to be the same for all junctions, whereas $\boldsymbol{\nabla}_\alpha\varphi_{\mathbf{n}}$ denotes the difference of $\varphi_{\mathbf{n}}$ between the neighboring sites of the lattice:

$$\boldsymbol{\nabla}_\alpha\varphi_{\mathbf{n}}\equiv\varphi_{\mathbf{n}+\mathbf{e}_\alpha}-\varphi_{\mathbf{n}}. \tag{19}$$

Notice that the combination

$$\theta_{\mathbf{n}\alpha}\equiv\boldsymbol{\nabla}_\alpha\varphi_{\mathbf{n}}-A_{\mathbf{n}\alpha}, \tag{20}$$

which enters as the argument of the Josephson energy $E_J(\theta)=-J\cos\theta$ is a gauge-invariant quantity.

The second term in Eq. (15) is the energy of the magnetic field

$$E_{\mathrm{mf}}=\frac{1}{2\mu_0}\int d^3\mathbf{r}\,\mathbf{B}^2(\mathbf{r}) \tag{21}$$

expressed in terms of the variables $A_{\mathbf{n}\alpha}$. The matrix $M_{\mathbf{nk}}\equiv M(\mathbf{n}-\mathbf{k})$ is usually called the mutual inductance matrix[19–21] and

$$(\boldsymbol{\nabla}\times A)_{\mathbf{n}}\equiv\sum_{\alpha,\beta}\epsilon_{\beta\alpha}\boldsymbol{\nabla}_\beta A_{\mathbf{n}\alpha} \tag{22}$$

is the directed sum of the variables $A_{\mathbf{n}\alpha}$ along the perimeter of a lattice plaquette (the lattice equivalent of $\mathrm{rot}\,\mathbf{A}$) and is proportional to the magnetic flux penetrating this plaquette.

Variation of Eq. (15) with respect to $A_{\mathbf{n}\alpha}$ gives the equation

$$I_{\mathbf{n}\alpha}=\frac{2e}{\hbar}\sum_{\mathbf{k}}\tilde{\boldsymbol{\nabla}}\times M^{-1}_{\mathbf{nk}}(\boldsymbol{\nabla}\times A)_{\mathbf{k}}, \tag{23}$$

which relates the value of the superconducting current in a junction

$$I_{\mathbf{n}\alpha}=I_c\sin(\boldsymbol{\nabla}_\alpha\varphi_{\mathbf{n}}-A_{\mathbf{n}\alpha}) \tag{24}$$

with the vector potential of the magnetic field induced by the presence of the currents in the array. Here [like in Eq. (22)] $\tilde{\boldsymbol{\nabla}}\times$ stands for $\Sigma_\beta\epsilon_{\beta\alpha}\tilde{\boldsymbol{\nabla}}_\beta$, whereas $\tilde{\boldsymbol{\nabla}}_\beta$ designates the lattice difference, analogous to the one defined by Eq. (19), but shifted in the negative direction

$$\tilde{\boldsymbol{\nabla}}_\beta X_{\mathbf{n}}\equiv X_{\mathbf{n}}-X_{\mathbf{n}-\mathbf{e}_\beta}. \tag{25}$$

On the other hand, variation of Eq. (15) with respect to $\varphi_{\mathbf{n}}$ gives the current conservation equation

$$(\tilde{\boldsymbol{\nabla}}I)_{\mathbf{n}}=0, \tag{26}$$

where

$$(\tilde{\boldsymbol{\nabla}}I)_{\mathbf{n}}\equiv\sum_\alpha[I_{\mathbf{n}\alpha}-I_{(\mathbf{n}-\mathbf{e}_\alpha)\alpha}] \tag{27}$$

is the lattice equivalent of divergence. Equation (26) can be alternatively obtained by the application of the operator $\tilde{\boldsymbol{\nabla}}_\alpha$ to Eq. (23). Therefore, Eqs. (23) and (26) [both obtained by variation of Eq. (15)] are not independent of each other.

The vector potential of the magnetic field created by the currents flowing in the array can be chosen purely transverse $(\mathrm{div}\,\mathbf{A}=0)$, which in terms of $A_{\mathbf{n}\alpha}$ corresponds to

$$\tilde{\boldsymbol{\nabla}}_\alpha A_{\mathbf{n}\alpha}=0. \tag{28}$$

In that case, Eq. (23) is reduced to

$$I_{\mathbf{n}\alpha}=\frac{2e}{\hbar}\sum_{\mathbf{k}}M^{-1}_{\mathbf{nk}}(-\boldsymbol{\Delta}_L A_\alpha)_{\mathbf{k}}, \tag{29}$$

where $\Delta_L\equiv\Sigma_\beta\tilde{\boldsymbol{\nabla}}_\beta\boldsymbol{\nabla}_\beta$ is the two-dimensional lattice analog of the Laplacian

$$(\Delta_L X)_{\mathbf{k}}=\sum_\beta(X_{\mathbf{k}+\mathbf{e}_\beta}-2X_{\mathbf{k}}+X_{\mathbf{k}-\mathbf{e}_\beta}). \tag{30}$$

Comparison of Eq. (29) with Eq. (5) allows us to find that for $|\mathbf{n}-\mathbf{k}|\gg1$

$$M^{-1}_{\mathbf{nk}}\approx\left(\frac{\hbar}{2e}\right)^2\frac{1}{\pi\mu_0 a|\mathbf{n}-\mathbf{k}|}. \tag{31}$$

On the other hand, for $|\mathbf{n}-\mathbf{k}|\sim1$ the form of $M^{-1}_{\mathbf{nk}}$ depends on the particular shape of superconducting islands.[21]

Linearization of Eqs. (23) and their solution allows us to show that when the magnetic fields of the currents in the array are taken into account, the logarithmic interaction of vortices becomes screened[19] at so-called magnetic-field penetration length Λ, exactly as it happens in superconducting films.[22] When screening is relatively weak (that is when $\Lambda\gg a$), the value of Λ is given by

$$\Lambda\approx\frac{2}{\mu_0 J}\left(\frac{\hbar}{2e}\right)^2 \tag{32}$$

and does not depend on the shape of superconducting islands forming the array.[19]

Instead of considering Hamiltonian (15) as dependent on two different types of variables defined on the sites $(\varphi_{\mathbf{n}})$ and on the bonds $(A_{\mathbf{n}\alpha})$ of the lattice, it is convenient to use a single variable, namely, the gauge-invariant phase difference $\theta_{\mathbf{n}\alpha}$ defined by Eq. (20). In terms of $\theta_{\mathbf{n}\alpha}$, the Hamiltonian (15) can be rewritten as

$$H=-J\sum_{\mathbf{n},\alpha}\cos\theta_{\mathbf{n}\alpha}+\frac{1}{2}\sum_{\mathbf{n},\mathbf{k}}(\nabla\times\theta)_{\mathbf{n}}M^{-1}_{\mathbf{nk}}(\nabla\times\theta)_{\mathbf{k}}, \tag{33}$$

variation of which with respect to $\theta_{\mathbf{n}\alpha}$ reproduces Eq. (23) in the form

$$I_{\mathbf{n}\alpha}=-\frac{2e}{\hbar}\sum_{\mathbf{k}}\tilde{\boldsymbol{\nabla}}\times M^{-1}_{\mathbf{nk}}(\boldsymbol{\nabla}\times\theta)_{\mathbf{k}}, \tag{34}$$

where the expression for the superconducting current

$$I_{\mathbf{n}\alpha}=I_c\sin\theta_{\mathbf{n}\alpha} \tag{35}$$

is naturally consistent with Eq. (24). As previously, the current conservation Eq. (26) can be obtained by application of the operator $\tilde{\boldsymbol{\nabla}}_\alpha$ to Eq. (34).

IV. DYNAMIC FLUCTUATIONS IN ARRAY OF RESISTIVELY SHUNTED JUNCTIONS

The dynamic description of the same system requires us to complement the Hamiltonian H by the dissipative function W (we assume that the array is overdamped and therefore its dynamics is purely relaxational). In the case of the array formed by SNS (superconductor–normal metal–superconductor) junctions one can describe dissipation in terms of the effective resistance shunting each junction (so-called RSJ model). This corresponds to $W\{\theta\}$ of the form

$$W=\eta\sum_{\mathbf{n},\alpha}\left(\frac{\partial}{\partial t}\theta_{\mathbf{n}\alpha}\right)^2, \tag{36}$$

where the effective viscosity

$$\eta=\left(\frac{\hbar}{2e}\right)^2\frac{1}{R} \tag{37}$$

is determined by the value of the shunting resistance R, which is assumed to be the same for all junctions. For W of the form (36), the conservation of energy is achieved when the time evolution of the variables $\theta_{\mathbf{n}\alpha}$ is governed by the standard equations of relaxational dynamics

$$\eta\frac{\partial}{\partial t}\theta_{\mathbf{n}\alpha}=-\frac{\partial H}{\partial\theta_{\mathbf{n}\alpha}}. \tag{38}$$

On the other hand, Eq. (38) can be rewritten in the form (34), where the expression for the current should be replaced by

$$I_{\mathbf{n}\alpha}=I_c\sin\theta_{\mathbf{n}\alpha}+\frac{\hbar}{2eR}\frac{\partial}{\partial t}\theta_{\mathbf{n}\alpha}. \tag{39}$$

The time derivative of $\theta_{\mathbf{n}\alpha}$ being proportional to the voltage, the second term in Eq. (39) can be easily identified as the normal current flowing in the junction. Consideration of purely relaxational dynamics means that we are neglecting capacitive effects and currents have to be conserved on each site of the lattice (in other words, only transverse currents are allowed). This is ensured by the form of Eq. (34), substitution of which into Eq. (26) automatically leads to its fulfillment for any form of $I_{\mathbf{n}\alpha}$.

In the presence of thermal fluctuations the right-hand side of Eq. (38) should be complemented with the random force term $\xi_{\mathbf{n}\alpha}(t)$

$$\eta\frac{\partial}{\partial t}\theta_{\mathbf{n}\alpha}=-\frac{\partial H}{\partial\theta_{\mathbf{n}\alpha}}+\xi_{\mathbf{n}\alpha}+f_{\mathbf{n}\alpha}, \tag{40}$$

the correlations of which are Gaussian and satisfy

$$\langle\xi_{\mathbf{n}\alpha}(t)\xi_{\mathbf{k}\beta}(t')\rangle=2\eta T\delta_{\mathbf{nk}}\delta_{\alpha\beta}\delta(t-t'), \tag{41}$$

where T is the temperature expressed in energy units (that is multiplied by the Boltzmann constant k_B). We also have included in the right-hand side of Eq. (40) the nonrandom external force $f_{\mathbf{n}\alpha}$ (to be discussed later).

In terms of the expression for the current the random force $\xi_{\mathbf{n}\alpha}$ corresponds to the appearance in Eq. (39) of the additional (fluctuating) contribution to normal current $\delta I_{\mathbf{n}\alpha}$

$$I_{\mathbf{n}\alpha}=I_c\sin\theta_{\mathbf{n}\alpha}+\frac{\hbar}{2eR}\frac{\partial}{\partial t}\theta_{\mathbf{n}\alpha}+\delta I_{\mathbf{n}\alpha}, \tag{42}$$

where $\delta I_{\mathbf{n}\alpha}\equiv-(2e/\hbar)\xi_{\mathbf{n}\alpha}$. Note that since Eq. (34) describes the relation between the currents and the magnetic field induced by them, it has to remain fulfilled also when the fluctuations of currents are taken into account. The validity of the current conservation Eqs. (26) remains ensured by the form of the right-hand side of Eq. (34).

The suggestion to describe the dynamics of a resistively shunted Josephson-junction array by Eqs. (42) has been put forward by Shenoy,[23] who did not consider fluctuations of the magnetic field, that is assumed $\theta_{\mathbf{n}\alpha}\equiv\varphi_{\mathbf{n}+\mathbf{e}_\alpha}-\varphi_{\mathbf{n}}$. In that case, substitution of Eqs. (42) into the current conservation Eqs. (26) leads to the dynamic equations for $\varphi_{\mathbf{n}}$ with nonlocal effective viscosity.[23] Quite remarkably, the inclusion into consideration of the magnetic-field fluctuations leads to a simplification of the dynamic equations which (in terms of $\theta_{\mathbf{n}\alpha}$) become local. The idea that in the presence of magnetic field fluctuations a resistively shunted Josephson-junction array can be described by Eqs. (40), where $\theta_{\mathbf{n}\alpha}$ is the gauge invariant phase difference, has been introduced by Domínguez and José.[20]

V. FLUCTUATION-DISSIPATION THEOREM

It is well known that when the time evolution of some variables $\{\theta\}$ is governed by the standard Langevin equations of the form (40), the equilibrium (that is calculated for $f_{\mathbf{n}\alpha}=0$) correlation function

$$C_{\mathbf{n}\alpha,\mathbf{k}\beta}(t-t')=\langle\theta_{\mathbf{n}\alpha}(t)\theta_{\mathbf{k}\beta}(t')\rangle_{f=0} \tag{43}$$

is related with the response function

$$G_{\mathbf{n}\alpha,\mathbf{k}\beta}(t-t')=\left.\frac{\delta\langle\theta_{\mathbf{n}\alpha}\rangle}{\delta f_{\mathbf{k}\beta}}\right|_{f=0} \tag{44}$$

by the fluctuation-dissipation theorem

$$G_{\mathbf{n}\alpha,\mathbf{k}\beta}(t)-G_{\mathbf{n}\alpha,\mathbf{k}\beta}(-t)=-\frac{1}{T}\frac{\partial}{\partial t}C_{\mathbf{n}\alpha,\mathbf{k}\beta}(t). \tag{45}$$

However, in a practical situation one is interested not in the response of the gauge-invariant phase difference $\theta_{\mathbf{n}\alpha}$ to the (unspecified) conjugate force $f_{\mathbf{n}\alpha}$, but rather in more readily observable quantities such as conductivity, which is the response of a current to the application of electric field. In a situation when electric field $\mathbf{E}(\mathbf{r})$ is created due to the presence of ac magnetic field, it is given by

$$\mathbf{E}(\mathbf{r})=-\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}), \tag{46}$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential defining the (total) magnetic induction $\mathbf{B}(\mathbf{r})=\mathrm{rot}\,\mathbf{A}(\mathbf{r})$.

In the presence of the external magnetic field $\mathbf{H}(\mathbf{r})=(1/\mu_0)\mathrm{rot}\,\mathbf{A}^{\mathrm{ext}}(\mathbf{r})$ the expression (21) describing the magnetic-field energy should be replaced by the expression for the Gibbs free energy

$$F_{\mathrm{mf}}=\int d^3\mathbf{r}\left[\frac{1}{2\mu_0}\mathbf{B}^2(\mathbf{r})-\mathbf{B}(\mathbf{r})\mathbf{H}(\mathbf{r})\right], \tag{47}$$

a variation of which in the absence of any other terms gives $\mathbf{B}(\mathbf{r})=\mu_0\mathbf{H}(\mathbf{r})$. This leads to the appearance in the Hamiltonian (33), describing the array, of the additional term

$$\Delta H=\sum_{\mathbf{n},\mathbf{k}}(\nabla\times\theta)_{\mathbf{n}}M^{-1}_{\mathbf{nk}}(\nabla\times A^{\mathrm{ext}})_{\mathbf{k}}. \tag{48}$$

Here, the variables

$$A^{\mathrm{ext}}_{\mathbf{n}\alpha}=\frac{2e}{\hbar}\int_{a\mathbf{n}}^{a(\mathbf{n}+\mathbf{e}_\alpha)}d\mathbf{r}\,\mathbf{A}^{\mathrm{ext}}(\mathbf{r}), \tag{49}$$

defined on lattice bonds, are related to the vector potential $\mathbf{A}^{\mathrm{ext}}(\mathbf{r})$ of the external magnetic-field exactly in the same way as earlier introduced variables $A_{\mathbf{n}\alpha}$ are related to the total vector potential $\mathbf{A}(\mathbf{r})$.

The form of the correction to the Hamiltonian given by Eq. (48) corresponds to the presence in Eq. (40) of the external force term

$$f_{\mathbf{n}\alpha}=-\sum_{\mathbf{k}}\tilde{\boldsymbol{\nabla}}\times M^{-1}_{\mathbf{nk}}(\boldsymbol{\nabla}\times A^{\mathrm{ext}})_{\mathbf{k}}. \tag{50}$$

Comparison of Eq. (50) with Eq. (34) shows that (up to the factor of $2e/\hbar$) $f_{\mathbf{n}\alpha}$ is related to $A^{\mathrm{ext}}_{\mathbf{n}\alpha}$ exactly in the same way as $I_{\mathbf{n}\alpha}$ is related to $\theta_{\mathbf{n}\alpha}$. This allows us to conclude that the correlation functions of the currents in the array

$$C^I_{\mathbf{n}\alpha,\mathbf{k}\beta}(t-t')\equiv\langle I_{\mathbf{n}\alpha}(t)I_{\mathbf{k}\beta}(t')\rangle \tag{51}$$

and the response function

$$G^I_{\mathbf{n}\alpha,\mathbf{k}\beta}(t-t')\equiv\left.\frac{\delta\langle I_{\mathbf{n}\alpha}\rangle}{\delta(A^{\mathrm{ext}}_{\mathbf{k}\beta})^t}\right|_{f=0} \tag{52}$$

have to satisfy the relation

$$G^I_{\mathbf{n}\alpha,\mathbf{k}\beta}(t)-G^I_{\mathbf{n}\alpha,\mathbf{k}\beta}(-t)=-\frac{\hbar}{2e}\frac{1}{T}\frac{\partial}{\partial t}C^I_{\mathbf{n}\alpha,\mathbf{k}\beta}(t) \tag{53}$$

completely analogous to Eq. (45). Here, $(A^{\mathrm{ext}}_{\mathbf{k}\beta})^t$ is the transverse part of $A^{\mathrm{ext}}_{\mathbf{k}\beta}$. As can be seen from the right-hand side of Eq. (50), the longitudinal part of $A^{\mathrm{ext}}_{\mathbf{k}\beta}$ is completely decoupled from fluctuations of $\theta_{\mathbf{n}\alpha}$.

In terms of the effective conductivity $g^{\mathrm{eff}}_{\mathbf{n}\alpha,\mathbf{k}\beta}$, defined as the response function

$$g^{\mathrm{eff}}_{\mathbf{n}\alpha,\mathbf{k}\beta}(t-t')\equiv\left.\frac{\delta}{\delta V^{\mathrm{ext}}_{\mathbf{k}\beta}}\langle I_{\mathbf{n}\alpha}\rangle\right|_{V^{\mathrm{ext}}=0} \tag{54}$$

of the current with respect to external voltage

$$V^{\mathrm{ext}}_{\mathbf{k}\beta}=-\frac{\hbar}{2e}\frac{\partial}{\partial t}(A^{\mathrm{ext}}_{\mathbf{k}\beta})^t, \tag{55}$$

Eq. (53) can be rewritten as

$$C^I_{\mathbf{n}\alpha,\mathbf{k}\beta}(t)=T[g^{\mathrm{eff}}_{\mathbf{n}\alpha,\mathbf{k}\beta}(t)+g^{\mathrm{eff}}_{\mathbf{n}\alpha,\mathbf{k}\beta}(-t)]. \tag{56}$$

For wave vectors small in comparison with $1/a$, the variables $I_{\mathbf{n}\alpha}$ can be identified with $a\mathbf{i}(\mathbf{x})$, whereas $V^{\mathrm{ext}}_{\mathbf{n}\alpha}$ with $a\mathbf{E}^{\mathrm{ext}}_{\parallel}(\mathbf{x})$, where $\mathbf{E}^{\mathrm{ext}}_{\parallel}(\mathbf{x})$ is the projection of the external electric-field $\mathbf{E}^{\mathrm{ext}}(\mathbf{x})$ on the plane $x_3=0$ [here and below we assume that $\mathbf{E}^{\mathrm{ext}}_{\parallel}(\mathbf{q},\omega)$ is transverse]. This allows us to rewrite Eq. (56) as

$$\langle\mathbf{i}^t\mathbf{i}^t\rangle_{\mathbf{q}\omega}=2T\,\mathrm{Re}[g^{\mathrm{eff}}(\mathbf{q},\omega)], \tag{57}$$

where the Fourier transform $g^{\mathrm{eff}}(\mathbf{q},\omega)$ of the effective conductivity is the coefficient of proportionality in the relation

$$\mathbf{i}(\mathbf{q},\omega)=g^{\mathrm{eff}}(\mathbf{q},\omega)\mathbf{E}^{\mathrm{ext}}_{\parallel}(\mathbf{q},\omega). \tag{58}$$

One should distinguish between $g^{\mathrm{eff}}(\mathbf{q},\omega)$ and (also momentum- and frequency-dependent) sheet conductance $g_{\square}(\mathbf{q},\omega)$, which is defined as the coefficient of proportionality between $\mathbf{i}(\mathbf{q},\omega)$ and *total* electric field $\mathbf{E}_{\parallel}(\mathbf{q},\omega)$,

$$\mathbf{i}(\mathbf{q},\omega)=g_{\square}(\mathbf{q},\omega)\mathbf{E}_{\parallel}(\mathbf{q},\omega). \tag{59}$$

The form of the current induced correction to $\mathbf{E}^{\mathrm{ext}}_{\parallel}(\mathbf{q},\omega)$ can be easily found with the help of Eqs. (5) and (46), which lead to

$$\mathbf{E}_{\parallel}(\mathbf{q},\omega)=\mathbf{E}^{\mathrm{ext}}_{\parallel}(\mathbf{q},\omega)+i\omega\frac{\mu_0}{2q}\mathbf{i}(\mathbf{q},\omega). \tag{60}$$

Substitution of Eq. (59) into Eq. (60) then gives

$$\mathbf{E}_{\parallel}(\mathbf{q},\omega)=\frac{1}{1-i\omega\dfrac{\mu_0}{2q}g_{\square}(\mathbf{q},\omega)}\mathbf{E}^{\mathrm{ext}}_{\parallel}(\mathbf{q},\omega) \tag{61}$$

and, accordingly,

$$g^{\mathrm{eff}}(\mathbf{q},\omega)=\frac{1}{-i\omega\dfrac{\mu_0}{2q}+Z_{\square}(\mathbf{q},\omega)}, \tag{62}$$

where $Z_{\square}(\mathbf{q},\omega)\equiv1/g_{\square}(\mathbf{q},\omega)$ is the momentum- and frequency-dependent sheet impedance.

The form of Eq. (62) suggests that the response of a current in a two-dimensional system to the external electric field is the same as if the proper sheet impedance of a system $Z_{\square}(\mathbf{q},\omega)$ has been connected in series with the other contribution, which can be considered as the effective impedance of the empty space surrounding this system. This additional contribution is purely inductive and corresponds to

$$L_s(q)=\frac{\mu_0}{2q}. \tag{63}$$

In the case of a superconducting system in the low-frequency limit $Z_\square(\mathbf{q},\omega)\approx -i\omega L_\square$, where $L_\square$ is the effective sheet inductance, substitution of which into Eq. (61) allows us to rewrite it as

$$\mathbf{E}_\parallel(\mathbf{q},\omega)=\frac{\Lambda q}{1+\Lambda q}\mathbf{E}_\parallel^{\rm ext}(\mathbf{q},\omega), \tag{64}$$

where

$$\Lambda=\frac{2L_\square}{\mu_0} \tag{65}$$

is the (two-dimensional) magnetic-field penetration length[22] already discussed in Sec. III. Since the transverse electric field appears only as a consequence of the ac magnetic field, the same length describes as well the screening of the transverse electric field.

The form of the fluctuation-dissipation theorem obtained after substitution of Eq. (62) into Eq. (57),

$$\langle \mathbf{i}^t\mathbf{i}^t\rangle_{\mathbf{q}\omega}=2T\,{\rm Re}\,\frac{1}{-i\omega L_s(q)+Z_\square(q,\omega)} \tag{66}$$

being completely independent of the details of the structure of the particular system used for its derivation, one can expect it to be valid also for other two-dimensional superconducting (or simply conducting) systems, in particular, superconducting films.

VI. RESULTS AND DISCUSSION

The substitution of Eq. (66) into Eq. (14) gives the expression for the flux noise power spectrum

$$S(\omega)=\frac{\mu_0^2T}{2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}F^2(q){\rm Re}\left[\frac{1}{-i\omega L_s(q)+Z_\square(q,\omega)}\right], \tag{67}$$

which is the central result of this work. It allows us, instead of constructing special theories explaining frequency dependence of $S(\omega)$ in different regimes, to relate it with the properties of $Z_\square(q,\omega)$.

It is of interest to compare Eq. (67) with the expression for the quantity [the correction to frequency-dependent mutual impedance $Z_m(\omega)$], which is measured in the framework of the two-coil method and is defined as the ratio of the measured voltage to the driving current. In our notation, this expression[10] can be rewritten as

$$\delta Z_m(\omega)=-\omega^2\frac{\mu_0^2}{4}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{F(q)\tilde F(q)}{-i\omega L_s(q)+Z_\square(q,\omega)}, \tag{68}$$

where $\tilde F(q)$ is the geometrical factor analogous to $F(q)$, but describing the driving coil. Note that the applicability of Eqs. (67) and (68) for the description of a Josephson-junction array requires the lattice constant of the array a to be much smaller then the geometrical parameters of the coil(s) (r and h) entering the expressions for $F(q)$ and $\tilde F(q)$.

Comparison of Eq. (68) with Eq. (67) shows that the real part of $\delta Z_m(\omega)$ is determined by the same (real) component of $g^{\rm eff}(q,\omega)$ as the noise spectrum, so in the absence of a difference between $F(q)$ and $\tilde F(q)$ the noise spectrum $S(\omega)$ would be simply proportional to ${\rm Re}[\delta Z_m(\omega)]$

$$S(\omega)=-\frac{2T}{\omega^2}{\rm Re}[\delta Z_m(\omega)]. \tag{69}$$

In the case of the simplest coil (a single circular loop of radius r) Eq. (67) is reduced to

$$S(\omega)=\pi\mu_0^2r^2T\int_0^\infty dq\left\{\frac{J_1^2(rq)}{q}\exp(-2hq)\times{\rm Re}\left[\frac{1}{-i\omega L_s(q)+Z_\square(q,\omega)}\right]\right\}. \tag{70}$$

The analogous equation derived by Kim and Minnhagen[17] in the framework of a less general approach (by expressing all quantities in terms of the vortex gas dielectric function) differs from Eq. (70) basically (i) by the absence of the term $L_s(q)$ and (ii) by the presence in the integrand of the additional factor q^4. The former of the two discrepancies is rather natural, since in Ref. 17 the screening effects have not been taken into account, whereas the latter we believe to be the consequence of the incorrect calculation of the magnetic field produced by the currents in the array.

In Ref. 17, this magnetic field is calculated as the sum of the magnetic fields produced by the current loops associated with lattice plaquettes, whereas the magnitude of a current in each loop is assumed to be given the directed sum of the currents in the junctions $I_{\mathbf{n}\alpha}$ along the perimeter of a plaquette (in the present paper this sum is denoted $\boldsymbol{\nabla}\times I$). In such a procedure the current of each junction is counted twice (as giving contributions to the loop currents associated with the two neighboring lattice plaquettes) but with opposite signs, so these two contributions almost cancel each other, which leads to the appearance of the additional q-dependent factor in comparison with our Eq. (70). In a more consistent implementation of this approach, the values of the loop currents associated with arrays plaquettes should be chosen in such a way that the value of the current on each junction is given by the difference of the loop currents associated with the two neighboring plaquettes. These so-called mesh currents[21,24] $I_{\mathbf{n}}^{\rm m}$ are related with the currents in the junctions $I_{\mathbf{n}\alpha}$ as

$$I_{\mathbf{n}}^{\rm m}\equiv-\Delta_L^{-1}(\boldsymbol{\nabla}\times I)_{\mathbf{n}}, \tag{71}$$

which explains the appearance of the additional factor of q^4 in calculation which uses $(\boldsymbol{\nabla}\times I)_{\mathbf{n}}$ instead of $I_{\mathbf{n}}^{\rm m}$. An analogous mistake is incorporated in numerical calculations of Ref. 17.

The main contribution to the integral in Eq. (70) is coming from the region

$$0<q\lesssim 1/h, \tag{72}$$

so in the situation when h exceeds all the microscopic scales responsible for the q dependence of $Z_{\square}(q,\omega)$, one can replace in Eq. (70) $Z_{\square}(q,\omega)$ by

$$Z_{\square}(\omega)\equiv -i\omega L_{\square}(\omega)+R_{\square}(\omega)=\lim_{q\to 0} Z_{\square}(q,\omega) \tag{73}$$

and use $S(\omega)$ to extract information about $L_{\square}(\omega)$ and $R_{\square}(\omega)$.

Let us first discuss the limit when the effects of screening can be neglected. This is possible if in the essential part of the interval (72) one can neglect $L_s(q)$ in comparison with $L_{\square}(\omega)$ and requires $\Lambda \gg h,\ r$ (and not $\Lambda \ll h$, as has been claimed in Refs. 4 and 17). In that case, Eq. (70) is reduced to

$$S(\omega)=\pi\mu_0^2 r^2 T Y(h/r)\frac{R_{\square}(\omega)}{\omega^2 L_{\square}^2(\omega)+R_{\square}^2(\omega)}, \tag{74}$$

where

$$Y(u)=\int_0^\infty dp\,\frac{J_1^2(p)}{p}\exp(-2up)\approx\begin{cases}1/2 & \text{for}\quad u\ll 1\\ 1/16u^2 & \text{for}\quad u\gg 1.\end{cases} \tag{75}$$

This means that for $h\ll r$ the amplitude of the noise has to be almost independent of h, whereas for $h\gg r$ it has to decay proportionally to $1/h^2$. The experimental results of Festin *et al.*,[4] obtained on thin YBCO film at $h/r\sim 1$, are compatible with $1/h^2$ dependence even better than with $1/h^3$ dependence erroneously predicted in Ref. 17 for $h\ll r$.

The frequency dependence of $S(\omega)$ in the weak screening regime is directly determined by the frequency dependence of $Z_{\square}(\omega)$. According to the results of Ambegaokar *et al.*,[25] below the temperature $T_{\rm BKT}$ of the Berezinskii-Kosterlitz-Thouless phase transition,[26,27] that is in the quasiordered phase of the array, the contribution to $R_{\square}(\omega)$ coming from vortex pairs has to be (for low-enough frequencies) of the algebraic form

$$R_{\square}^{\rm vp}(\omega)\propto\omega^{2K(T)-1}, \tag{76}$$

where $K(T)$ is the prelogarithmic factor in the vortex-vortex interaction divided by $4T$. With increasing temperature, $K(T)$ monotonously decreases from $\pi J(T)/2T$ at $T\ll J(T)$ to 1 at $T=T_{\rm BKT}$, demonstrating on approaching $T_{\rm BKT}$ a square-root singularity

$$K(T)-1\propto\sqrt{\frac{T_{\rm BKT}-T}{T_{\rm BKT}}}. \tag{77}$$

Substitution of Eq. (76) and $L_{\square}(\omega)\approx$ const [below $T_{\rm BKT}$, the main contribution to $L_{\square}(\omega)$ is purely superconductive] into Eq. (74) gives that in the absence of screening the noise spectrum associated with vortex pairs should be of the form

$$S(\omega)\propto\omega^{2K(T)-3}. \tag{78}$$

Above $T_{\rm BKT}$, the presence of free vortices leads to a finite zero-frequency resistance [proportional to free vortex concentration $n_{\rm v}$ (Ref. 25,28)] and, therefore, to frequency independent (at low frequencies) noise $S(\omega)\propto T/n_{\rm v}$.

Experimental data of Refs. 7 and 8 (taken presumably above $T_{\rm BKT}$) indeed demonstrates a crossover from almost white noise at low frequencies to the noise with algebraic frequency dependence at higher frequencies. The value of $S(\omega=0)$ increases with deceasing temperature in qualitative agreement with $S(\omega)$ being inversely proportional to $n_{\rm v}(T)$. However, the algebraic dependence of $S(\omega)$ on ω observed at higher frequencies (and at lower temperatures) cannot be unambiguously ascribed to vortex pairs contribution, because in contrast to Eq. (78) the value of the exponent describing this dependence $[S(\omega)\propto 1/\omega]$ remains independent of temperature in a wide interval of dimensionless temperatures $\tau\equiv T/J(T)$ and in terms of Eq. (78) corresponds to $T=T_{\rm BKT}$.

In the opposite limit of strong screening, $L_{\square}(\omega)$ can be neglected in comparison with $L_s(\omega)$, and $S(\omega)$ can depend only on $R_{\square}(\omega)$, but not on $L_{\square}(\omega)$. In particular, for $r\gg\Lambda,h$ integration in Eq. (70) gives

$$S(\omega)\approx\frac{\pi\mu_0 r T}{\omega} \tag{79}$$

for $\omega\ll R_{\square}(\omega)/\mu_0 h, R_{\square}(\omega)/\mu_0\Lambda$ and

$$S(\omega)\approx\frac{2r}{h}\,\frac{R_{\square}(\omega)T}{\omega^2} \tag{80}$$

for $\omega\gg R_{\square}(\omega)/\mu_0 h$ and $h\gg\Lambda$.

Notice that Eq. (79) corresponds to the very special regime of the universal $1/\omega$ noise, the amplitude of which does not depend on $Z_{\square}(\omega)$ and, accordingly, does not allow one to extract any information about the properties of the sample. It is, therefore, natural to inquire if (almost) temperature-independent $1/\omega$ noise observed in Refs. 7 and 8 over four decades in frequency can be interpreted as resulting from the realization of such a strong screening regime. The idea that $1/\omega$ dependence appears because the SQUID (superconducting quantum interference device) integrates contributions from a wide interval of wave vectors has been put forward by Wagenblast and Fazio.[14]

However, our derivation has shown that in order to obtain the $1/\omega$ dependence over four decades in frequency as a result of the integration over q in a strong screening regime one should have $r/h\gtrsim 10^4$, which definitely has not been fulfilled in the experiments of Refs. 7 and 8. Thus, the origin of the $1/\omega$ dependence of the flux noise power spectrum observed experimentally in Josephson-junction arrays still remains to be elucidated. The analogous frequency dependence of the noise spectrum in thin YBCO films[5] is ascribed to the vortex hopping between neighboring pinning centers, a finite vortex concentration being associated with the presence of a residual magnetic field. The same mechanism may be responsible for the results of the experiments[6–8] on Josephson-junction arrays.

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