sweepback angle $\psi$. Curve 1 corresponds to the wing with $\lambda = 1$, curve 2 to $\lambda = 0.75$, and curve 3 to $\lambda = 1.5$. The curves $c_{y\psi}/c_y(\psi)$ obtained in the calculations are similar.

On the basis of our investigations it can be concluded that if a delta wing of small aspect ratio has a positive transverse sweepback the vortices formed on the leading edges will break up at larger angles of attack than in the case of the original flat wing. In contrast, a negative transverse sweepback leads to breakup of the vortices at comparatively smaller angles of attack. This is more strongly pronounced for a wing with $\lambda = 1.5$ and to a lesser degree to wings with $\lambda = 1.0$ and 0.75.

In addition, both positive and negative transverse sweepback of delta wings lead in the majority of cases to an appreciable decrease in their carrying properties compared with the flat wing. The effect is greater for wings with positive transverse sweepback.

LITERATURE CITED


1. P. K. Chang, Separation of Flow, Pergamon, Oxford (1970).
2. W. H. Wentz and D. L. Kohlman, "Vortex breakdown on slender sharp-edged wings," AIAA Pap., No. 778, 15 (1969).
3. A. A. Karask and M. I. Nisht, "Vortex structures of slender wings in the case of flow separation," Dokl. Akad. Nauk SSSR, 246, 1317 (1979).
4. V. A. Aparinov, S. M. Belotserkovskii, M. I. Nisht, and O. N. Sokolova, "Mathematical simulation in an ideal fluid of flow separation from a wing and breakup of the vortex sheet," Dokl. Akad. Nauk SSSR, 227, 820 (1976).
5. S. M. Belotserkovskii and M. I. Nisht, Flow of an Ideal Fluid over Slender Wings with and without Separation [in Russian], Nauka, Moscow (1978), p. 351.
6. D. Hummel, "Untersuchungen über das Aufplatzen der Wirbel an Schlanken Delta-flügeln," Z. Flugwiss., 13, No. 5, 158 (1965).


# STABILITY OF SHOCK WAVES WITH FINITE RELAXATION REGION


S. E. Korshunov



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A theoretical investigation is made into the amplification of sound in a moving nonequilibrium medium and it is shown that an instability can arise in a sufficiently strong shock wave accompanied by an exothermic process with finite relaxation region, the instability being due to the spontaneous growth of fluctuations resulting from amplification of acoustic waves in the region of exothermic relaxation and their "trapping" in a narrow layer near the shock wave.


In the first theoretical investigation [1] of shock wave instability, the influence of relaxation physicochemical processes behind the wave was taken into account only indirectly as one of the possible reasons for an anomalous form of the Hugoniot adiabat; the relaxation layer was assumed to be thin.

The experimental observation of shock wave instability [2, 3] stimulated interest in the establishment of a more intimate connection between relaxation and shock wave instability [4, 5], though this was done phenomenologically.

## 1. Geometrical Acoustics of a Moving Relaxing Medium

To investigate the possibility of amplification of a sound wave propagating in a moving relaxing medium, we use the method proposed by Blokhintsev [6] to derive the basic equation of geometrical acoustics.

We consider a medium whose thermodynamic functions depend not only on the temperature T and the density $\rho$ but also on some third parameter $\zeta$. Suppose the change in $\zeta$


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is described by the ordinary equation

$$\frac{\partial}{\partial t}(\rho\zeta)+\operatorname{div}(\rho\zeta\mathbf{v})=\rho K(\rho, T, \zeta) \tag{1.1}$$

of chemical kinetics. Here, $\mathbf{v}$ is the velocity of the medium and the rate of the relaxation process is expressed for convenience in the form $\rho K$.

To construct an approximate theory of the propagation of sound, we represent all the small deviations that occur in the linearized Euler equations, the mass and energy conservation equations, and the linearized equation (1.1) in the form of products of slowly varying factors (the amplitudes) and a rapidly varying exponential factor

$$\exp[i(-\omega t+\Phi)].$$

Here, $\omega$ is the frequency and $\Phi$ is the time-independent part of the phase of the sound wave. The wave vector $\mathbf{k}$ of the wave is grad $\Phi$.

We shall assume that $\omega$ is large compared with the reciprocal characteristic time of the relaxation process (the acoustic approximation), which enables us to seek the amplitudes in the form of power series in inverse powers of $\omega$ (only the first two terms are required).

Substituting such expansions in the original linearized system, we can find that the conditions of compatibility of the system for the leading terms and the first correction terms of the expansions in $\omega^{-1}$ have, respectively, the form

$$(\omega-\mathbf{vk})^2=c^2k^2 \tag{1.2}$$

$$\frac{\partial}{\partial t}\varepsilon+\operatorname{div}(\mathbf{V}\varepsilon)=\lambda\varepsilon \tag{1.3}$$

Here, $\varepsilon$ is the energy density, $c^2 = (\partial p/\partial\rho)_{s,\zeta}$ is the square of the phase velocity, $\mathbf{V} = \mathbf{v} + c\mathbf{k}/k$ is the group velocity,

$$\lambda=-\left(\frac{\partial E}{\partial \zeta}\right)_{p,\rho}\left[\frac{c_v\rho^2}{p}\left(\frac{\partial K}{\partial \rho}\right)_{T,\zeta}+\left(\frac{\partial K}{\partial T}\right)_{\rho,\zeta}\right]\left[c_v\left(1+\frac{Tc_v\rho}{p}\right)\right]^{-1} \tag{1.4}$$

is the amplification coefficient of the sound wave, E is the internal energy density, p is the pressure of the medium, and $c_v = (\partial E/\partial T)_{p,\zeta}$.

Equation (1.2), which describes the propagation geometry of the sound waves, has exactly the same form as in the absence of the relaxation process. The leading terms in the expansions of the amplitudes are also unchanged, in particular, the amplitude $\zeta$ in the zeroth approximation is equal to zero. Therefore, the presence of the relaxation process does not influence the quantities that can be expressed in terms of the leading terms of the expansions, for example, the coefficient of reflection of the sound wave by the shock wave.

The presence on the right-hand side of Eq. (1.3) of a nonvanishing term means that the energy of an acoustic wave propagating in a relaxing medium varies. Amplification corresponds to $\lambda > 0$.

If we restrict ourselve to the most common case, when K increases sharply in modulus with increasing temperature and, therefore, the sign of the second factor in the expression (1.4) is equal to the sign of K, we note that under such an assumption the sign of $\lambda$ is determined by the direction of transformation of the energy in the relaxation process, namely, there is amplification of the sound in the presence of an exothermic process.

## 2. Steady Shock Wave in a Relaxing Medium

We use the following very simple model: the medium consists of an ideal gas (with specific heat ratio $\gamma$) with two energy levels separated by $\Delta > 0$. By $\zeta$ we denote the relative mass concentration of the molecules in the lower level. We shall assume that in front of the shock wave the gas is in a metastable state ($\zeta = 0$) due to the very low relaxation rate at its initial temperature $T_0$ and that the shock wave has sufficient amplitude to heat the gas so much that the relaxation process becomes important.

We choose a coordinate system such that the shock front is at rest in the yz plane and the gas moves in the positive direction of the x axis. We shall assume that the gas flow is steady and one dimensional.

It can be shown that the simultaneous solution of the Euler and Bernoulli equations, the equations of state and mass conservation, and the kinetic equation in the region x > 0 (behind the wave) has the form

$$v=v_1(1+u);\quad \rho=\rho_1(1+u)^{-1};\quad T=T_1(1+u)(1-\gamma M_1^2u),\quad \zeta=\frac{T_1}{\Delta}\frac{\gamma}{\gamma-1}u\left[(1-M_1^2)-\frac{\gamma+1}{2}M_1^2u\right] \tag{2.1}$$

Here, the subscript 1 is appended to the quantities in the limit $x \to +0$, M is the Mach number, and u is a parameter that is the dimensionless deviation of v from the initial value, its dependence on x being given implicitly by means of the integral

$$x=\frac{v_1}{(\gamma-1)\Delta}\int_0^u\frac{[\gamma T(u')-v^2(u')]\rho(u')}{K(\rho(u'),T(u'),\zeta(u'))}du \tag{2.2}$$

in which the functions (2.1) must be substituted.

It follows from the form of the integral (2.2) that when x increases from 0 to $+\infty$ the function u increases monotonically from 0 to $u_0$, the minimal positive root of the equation K(u) = 0.

For the following calculations, we also need the form of $\Lambda = c^2 - v^2$, expressed as a function of u:

$$\Lambda=\gamma T_1[(1-M_1^2)+(1-(\gamma+2)M_1^2)u-(\gamma+1)M_1^2u^2] \tag{2.3}$$

## 3. "Trapping" of Sound Waves

We consider the propagation of a sound wave in a medium behind a shock wave that excites a relaxation process. Because of the homogeneity of the medium with respect to the coordinates y and z, the components $k_y$ and $k_z$ of the wave vector of the sound wave will be conserved. We direct the y axis such $k_z$ is equal to zero and consider a sound wave with some given $k_y \neq 0$. Then, using (1.2), we can find $k_x$ at any point of the medium:

$$k_x=\frac{-\omega v\pm c(\omega^2-\Lambda k_y^2)^{1/2}}{c^2-v^2} \tag{3.1}$$

The two solutions correspond to the two different signs of the projection of the group velocity of the wave onto the x axis. At the point where the radicand vanishes, this projection also vanishes, and then reverses its direction, which corresponds to the sign change in front of the root in (3.1).

Thus, if the maximal value of $\Lambda$, $\Lambda_{max}$, on the interval x > 0 exceeds $\Lambda(0)$, any sound wave with frequency $\omega$ satisfying

$$\Lambda(0)k_y^2<\omega<\Lambda_{max}k_y^2 \tag{3.2}$$

that propagates away from the shock wave (at an angle to the x axis, since $k_y \neq 0$) returns to it. After reflection by the shock wave, it repeats the motion and, thus, is "trapped" in a certain layer near the shock front.

The condition (3.2) means that there exists a certain maximal value of the angle $\beta$ between the plane of the shock wave and the group velocity of the sound wave leaving it for which the effect is possible.

It follows from the form of the trinomial (2.3) that for the chosen model of a relaxing medium sound waves are "trapped" when

$$M_1^2<1/(2+\gamma) \tag{3.3}$$

This condition is equivalent to the following inequality for the temperatures of the medium behind and in front of the wave:

$$T_1>(2+\gamma)T_0/[3(2-\gamma)] \tag{3.4}$$

and it does not contradict the assumption previously made that there is a significant jump of the temperature at the shock wave. Note that in the case of an endothermic relaxation process the condition (3.4) for "trapping" of sound waves has the opposite sign of the inequality.

## 4. Unbounded Growth of the Amplitude of "Trapped" Sound Waves

Thus, it has been established that if a sufficiently strong shock wave accompanied by an exothermic relaxation process propagates in the medium sound waves that leave the shock wave at sufficiently small angles to the plane of the wave will return to the plane of the shock wave, having been amplified during the process. If the amplification is greater than the attenuation due to reflection from the shock wave, the amplitude of such sound waves, existing in the form of thermal fluctuations, will (in the linear approximation) increase unboundedly, leading to shock wave instability.

For an ideal gas, the amplification coefficient $\lambda$ will be

$$\lambda = \frac{\gamma-1}{\gamma T} BK\Delta; \qquad B = \frac{\gamma-1}{\gamma}\frac{T}{K}\left(\frac{\partial K}{\partial T}\right)_{\rho,\zeta} + \frac{\rho}{K}\left(\frac{\partial K}{\partial \rho}\right)_{T,\zeta} > 0 \tag{4.1}$$

If we restrict ourselves to the limiting case of small angles $\beta$, the considered sound waves will be "trapped" in a layer whose thickness is small compared with the characteristic distance over which the medium relaxes, which makes it possible to ignore the variations of some of the parameters of the medium when the amplification of the sound is calculated.

Integration of (1.3) for a steady one-dimensional flow with allowance for (4.1) shows that in the considered limiting case the relative change in the energy density of the sound wave during one period of its macroscopic motion is

$$F = \frac{1-M_1\alpha_1}{1+M_1\alpha_1} \exp\left[ B\frac{4M_1(1-M_1^2)}{1-(\gamma+2)M_1^2}\alpha_1 \right] \tag{4.2}$$

Here $\alpha = (1-\Lambda(u)k_y^2/\omega^2)^{1/2}$; $\alpha_1 = \alpha(0)$; for $\beta \ll 1$ $\beta \approx (1-M_1^2)\alpha_1$.

In accordance with the remark made in Sec. 1, the value of the coefficient of reflection R of the sound wave by the shock wave can be calculated without allowance for the relaxation process and for arbitrary $\alpha_1$ is

$$R = \frac{1-M_1\alpha_1}{1+M_1\alpha_1}\left(\frac{M_0^{-2}+\alpha_1^2-M_1\alpha_1}{M_0^{-2}+\alpha_1^2+M_1\alpha_1}\right)^2, \qquad M_0^{-2} = \frac{-(\gamma-1)+2\gamma M_1^2}{2+(\gamma-1)M_1^2} \tag{4.3}$$

The criterion for the occurrence of shock wave instability is the inequality FR > 1. It follows from (4.2) and (4.3) that in the case of small $\beta$ a sufficient condition for this is fulfillment of the inequality

$$B > \frac{[(3-\gamma)+(3\gamma-1)M_1^2][1-(\gamma+2)M_1^2]}{[2\gamma M_1^2-(\gamma-1)](1-M_1^2)}$$

which for any positive B will have a certain interval of solutions, which also satisfy the condition (3.3).

Thus, our calculation shows the possibility of existence of shock wave instability in the acoustic approximation by virtue of a relaxation process with release of energy. This agrees with the results of numerical calculation of a problem of detonation instability [7], which show that there are eigenvalues lying in the region of the acoustic approximation.

With regard to the occurrence of instability in the case of endothermic relaxation processes, for example, when there is excitation of new degrees of freedom, our approach only enables us to establish the absence of unstable eigenvalues in the region of validity of the acoustic approximation and does not establish whether they exist or not in the remaining part of the upper half-plane of values of the complex variable $\omega$.

It should be noted that in a real gas shock wave instability occurs in accordance with the mechanism we have described only if the amplification of the acoustic waves exceeds their absorption due to viscosity and heat conduction, which leads to a decrease or even the disappearance of the region of occurrence of the shock wave instability.

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LITERATURE CITED

1. S. P. D'yakov, "The stability of shock waves," Zh. Eksp. Teor. Fiz., 27, 288 (1954).
2. R. W. Griffiths, R. J. Sandeman, and H. G. Hornung, "The stability of shock waves in ionizing and dissociating gases," J. Phys. D: 9, 1681 (1976).
3. A. S. Baryshnikov, A. P. Bedin, V. G. Maslennikov, and G. I. Mishin, "Instability of the front of a bow shock," Pis'ma Zh. Tekh. Fiz., 5, 281 (1979).
4. A. S. Baryshnikov and G. E. Skvortsov, "Instability of shock waves in a relaxing medium," Zh. Tekh. Fiz., 49, 2483 (1979).
5. A. S. Baryshnikov, "Investigation of the dispersion relation in the problem of relaxation instability," Zh. Tekh. Fiz., 50, 1314 (1980).
6. D. I. Blokhintsev, "Propagation of sound in an inhomogeneous and moving medium," Dokl. Akad. Nauk SSSR, 45, 343 (1944).
7. V. V. Pukhnachev, "Stability of Chapman—Jouguet detonation," No. 6, 66 (1963).

# CALCULATION OF REFLECTION OF A BLAST WAVE BY A PLANE

V. A. Andrushchenko and Kh. S. Kestenboim

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Some results are given of calculation of the reflection of a blast wave by a rigid flat surface. A model of the explosion with a simple energy dissipation mechanism is considered, radiation being taken into account in the approximation of radiative heat conduction. The pressure distribution on the surface and the flow pattern in the region of propagation of the incident and reflected shock waves are obtained.

1. The solution to the problem of the interaction of a spherical shock wave with a plane is important, for example, in the analysis of the effects observed following explosions in the Earth's atmosphere of large meteoritic bodies [1, 2]. As a result of diffraction of a blast wave by a plane, a strong reflected wave is formed. Because of the curvature of the wave front, initially regular but then Mach reflection is observed at the point of contact of the incident shock with the surface [3]. The reflected wave, advancing through the disturbed medium, reaches the hot central region of the explosion and interacts with the region of sharp increase in the density. As a result, there is formed in the flow a system of secondary shocks [4] moving in the direction of the expanding bow shock and toward the plane. In particular, one can observe the arrival at the plane of a secondary shock wave with subsequent reflection. The flow pattern is significantly complicated by the time dependence and the interference of the waves in the disturbed region. It is evident that accurate calculation of the wave stage of the reflection process must introduce certain corrections in the distributions of the gas-dynamic parameters both in the region of the motion as well as on the surface. For example, in contrast to [3], the pressure on the surface behind the point of contact of the bow shock is not a monotonic function of the distance from the epicenter.

In [3, 4], calculations of the reflection were made on the basis of numerical integration of the equations of motion of an inviscid gas that does not conduct heat. As initial conditions, the solution to the problem of an explosion in a homogeneous atmosphere was used [5]; this has a singularity (infinite temperature and zero density) in the hot central region. This made it difficult to use standard difference methods to calculate the motion of the gas in the complete disturbed region. To overcome this difficulty, half of the disturbed region with the hot zone was eliminated from consideration in [3], while in [4] the temperature was artificially "cut off" at the center of the explosion. Evidently, a more promising approach is to give a more refined formulation of the problem with allowance for dissipative factors (viscosity, radiative

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