Coherent and incoherent tunneling in a Josephson junction with a "periodic" dissipation

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A tunnel junction shunted by a normal resistance is studied. The band width at zero external current and the tunneling probability as a function of the external current under incoherent tunneling conditions are found for the limiting case of a large viscosity.

According to Ambegaokar *et al.*,[1] a tunnel junction which is shunted by a normal resistance can be described by an effective Euclidean action

$$S[\varphi(t)] = \int dt \left[\frac{m}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - V\cos\varphi - F\varphi\right] + \frac{4\eta}{\pi}\int\int dt\,dt'\,\frac{\sin^2\{[\varphi(t)-\varphi(t')]/4\}}{(t-t')^2}, \tag{1}$$

which depends on a single variable φ, the phase difference at the junction. Here $m = \hbar C/4e^2$ (C is the capacitance of the junction) is the effective mass, $\eta = \hbar/4e^2R$ (R is the shunt resistance) is the effective viscosity, $V = I_c/2e$ (I_c is the critical current), and $F = I/2e$ (I is the external current). The interaction with the microscopic degrees of freedom is manifested in (1) as a term which is nonlocal with respect to time and which depends on $\varphi(t) - \varphi(t')$ periodically.

Guinea and Schön[2] studied the model (1) at $F = 0$ and showed that its large viscosity, in contrast with a similar system with a nonlocal dissipation quadratic in [3]$\varphi(t) - \varphi(t')$, does not lead to a complete localization: The wave function remains smeared either for all even or for all odd minima of the periodic potential, because the tunneling to the nearest minimum is suppressed completely, while the tunneling to the minimum next to the nearest minimum has a finite amplitude.

The approximate transformations used in Ref. 2 upon changing from (1) to an equivalent two-level system can be used only when $mV \gg \eta^2$, 1.

We will analyze below another part of the semiclassical approximation which is valid and which corresponds to the large-viscosity limit,

$$\eta >> (mV)^{1/2}, \ln(\eta^2/mV). \tag{2}$$

Apart from the formation of the band at $F = 0$, we will also consider its destruction by the external current and the transition to the incoherent tunneling. The temperature is assumed to be zero.

At $F = 0$ and $\eta \neq 0$ action (1) has not only an extremum that connects the neighboring minima of the potential (ordinary instanton) but also an extremum that connects the minima which are situated next to the neighboring minima (a double instanton). The amplitude of the tunneling to the minimum that comes after the nearest minimum is determined by the action along this path (whose magnitude is finite) and by the fluctuations in its neighborhood. The exact form of the extremum $\Phi(t)$ can be found only in the large-viscosity limit $(m, V=0)$:

$$\Phi(t)=4 \arctan \Omega t, \tag{3}$$

where Ω is arbitrary for the time being.

Substituting (3) in (1), we find

$$S[\Phi(t)]=4\pi(\eta+m\Omega+V/\Omega),$$

which means that $\Omega=(V/m)^{1/2}$ and $S_0\equiv S(\Omega)=4\pi\eta+8\pi(mV)^{1/2}$.

To evaluate the coefficient of the exponential function, we must find the eigenvalues of the operator $(\delta^2 S/\delta\varphi^2)_{\varphi=\Phi(t)}$. In the same approximation $(m, V=0)$ the equation for the eigenfunctions

$$\eta\left\{ \fint \frac{dt'}{\pi}\frac{1}{t-t'}\frac{\partial\tilde{\varphi}}{\partial t'}-\frac{2\Omega}{1+(\Omega t)^2}\left[\tilde{\varphi}(t)-\int\frac{dt'}{\pi}\frac{\Omega}{1+(\Omega t')^2}\tilde{\varphi}(t')\right]\right\}=\Lambda\tilde{\varphi}(t)$$

is essentially the same as that in the system with a quadratic dissipation for $m=0$, $V\neq 0$ (Ref. 4). This equation has the following solutions:

$$\tilde{\varphi}_0(t)=1/(1+\Omega^2t^2);\ \Lambda_0=0$$

$$\tilde{\varphi}_{\pm\epsilon}(t)=\exp[\pm i(\epsilon t+\arctan\Omega t)];\ \Lambda_{\pm\epsilon}=\eta\epsilon\quad(\epsilon\geqslant 0).$$

The displacement of the double instanton as a whole along the time scale corresponds to the mode $\tilde{\varphi}_0(t)$ and the change in the parameter Ω corresponds to the mode $\tilde{\varphi}_{+0}(t)-\tilde{\varphi}_{-0}(t)$.

Having estimated the shift in the eigenvalues for the finite values of m and V, we find, after a regularization, the amplitude Δ of the tunneling to the minimum of the potential that immediately follows the nearest minimum:

$$\Delta\sim\frac{2\eta^2}{(mV)^{3/4}\Omega}\exp(-S_0)$$

which determines the band width.

The interaction of the double instantons is inversely proportional to the square of the distance between them τ (with respect to the logarithmic interaction, they are dipoles). For $0<F\ll V$ the action along the double-instanton path which corresponds to the tunneling to a lower-lying minimum among those that follow the nearest minima is

$$S(\tau)=2S_0-16\pi c\eta(\Omega\tau)^{-2}-4\pi F\tau, \tag{4}$$

where $c\approx 2$ for $1\ll\Omega\tau\ll(\eta^2/mV)^{1/4}$ and $c\approx 1$ for $\Omega\tau\gg(\eta^2/mV)^{1/4}$. Action (4) reaches the extremum

$$S_* = 2S_0 - 12\pi(c\eta F^2/\Omega^2)^{1/3} \tag{5}$$

at $\tau_* = 2(c\eta/F\Omega^2)^{1/3} \gg \Omega^{-1}$ [here and below $c \equiv c(\tau_*)$].

If the condition $(\partial^2 S/\partial\tau^2)_{\tau=\tau_*} \gg \tau_*^{-2}$ is satisfied, i.e., if

$$F >> F_0 \sim [(24\pi)^3 \eta]^{-1/2}\, \Omega\ ,$$

the fluctuations around the extremal path, which we are considering here, will be small. This circumstance allows us to find from this path, in the exponential approximation, the probability for the tunneling to the minimum that follows the nearest minimum, which is an incoherent minimum in this case (a purely exponential relaxation):

$$P_2 = \Delta^2 (c\eta/27\Omega^2 F^4)^{1/3} \exp[\,12\pi(c\eta F^2/\Omega^2)^{1/3}]. \tag{6}$$

The quantity F_0 determines the current at which the band nature of the motion is destroyed and at which a transition to an incoherent tunneling occurs. For Eq. (6) to have a broad range of applicability: $F_0 \ll F \ll V$, the condition $F_0 \ll V$ must hold. As a result, an additional condition is imposed on η: $\eta \gg (24\pi)^{-3}(mV)^{-1}$, which is satisfied, however, in a large part of the region (2).

Because of the logarithmic nature of the interaction of the ordinary (single) instantons, at $F=0$ the tunneling to the nearest minimum is suppressed completely. At $F>0$ an incoherent tunneling occurs to a lower-lying minimum of the nearest minima with a probability $P_1 \propto F^{4\eta/\pi - 1}$. A comparison of the action along the extremal path with (5) shows that P_1 is equal to P_2 at $\ln(F/V) \sim -2\pi^2$ and is greater than P_2 at higher values of the external current. At typical values of the parameters we have $P_1 \gg P_2$ over the entire range of applicability of Eq. (6), greatly decreasing the possibility of an experimental study of incoherent tunneling to the minimum of the potential that lies next to the nearest minimum.

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