Quantum diffusion of vortices in 2D lattice systems

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The temperature dependence of the probability for an interstitial tunneling of vortices in an incoherent regime is derived for several lattice quantum models. The results can be used to describe 2D systems such as a planar ferromagnet, various lattices of tunnel junctions, and the free surface of a quantum crystal.

Vortices play a key role[1] in the thermodynamics of 2D systems with a continuous degeneracy of the order parameter in the O(2) group. The low-temperature (ordered) phase is distinguished from the high-temperature phase in that it has no free vortices (i.e., vortices which are not part of neutral pairs). Ambegaokar *et al.*[2] have studied the effect of vortices on the dynamic properties of both phases, working from the assumption that the motion of an isolated vortex is of a diffusive nature.

Research on the dynamic properties of regular 2D structures consisting of coupled tunnel junctions of superconducting elements has recently been intensifying (see, for example, Ref. 3 and the bibliography there). There is clear interest in the nature of the motion of vortices in lattice systems of one sort or another with a continuous degeneracy, in particular, in a context where it is possible to compare experimental data with theoretical predictions.[2] In the present letter we analyze the dynamics of vortices in an ordered phase for certain simple quantum models which have a variety of applications.

In the simplest approximation, a regular lattice of tunnel junctions can be described by the Hamiltonian

$$\hat{H} = \sum_{\mathbf{j}} \frac{J}{2} \hat{n}_{\mathbf{j}}^2 - \sum_{(\mathbf{j}\mathbf{j}')} V\cos(\phi_{\mathbf{j}} - \phi_{\mathbf{j}'});\ \hat{n}_{\mathbf{j}} \equiv -i\,\frac{\partial}{\partial \phi_{\mathbf{j}}}\,, \tag{1}$$

where $\phi_{\mathbf{j}}$ is the phase of granule $\mathbf{j}$, $J = 4e^2/C$ (C is the capacitance of a granule), $V = \hbar I_c/2e$ (I_c is the critical current of the isolated junction), and the summation in the last term runs over the pairs of nearest neighbors in the lattice. The same model describes a planar ferromagnet in the approximation of a continuous spin. We will consider here only the case $V \gg J$, in which the quantum fluctuations are small, and there is a true long-range order at zero temperature.

The distribution of the field ϕ which corresponds to a vortex (and in which the potential energy has a true local minimum) is centered in a particular lattice cell. The motion of the core of a vortex to a neighboring cell requires surmounting a barrier[4] $\Delta V \approx 0.40V$. At low temperatures this is a tunneling process. Lobb, Abraham, and Tinkham[4] attempted to study interstitial tunneling of vortices, but they assumed that the only essential difference between this process and phase tunneling at a single junction was a difference in the barrier height (this assumption is wrong). They applied to an isolated junction an equation which is incorrect, at least at low temperatures.

The exact distribution of the ϕ field in a vortex and on an extremal trajectory in imaginary time, which corresponds to the tunneling of a vortex between adjacent cells (we will call such trajectories "instantons"), can be found by replacing the potential $V(\Delta\phi) = -V\cos(\Delta\phi)$ in (1) by a periodically continued parabola $V(\Delta\phi) = (V/2)\min[\Delta\phi - 2\pi p)^2]$, where p is an integer. In the case of a square lattice (whose sites correspond to integer values of the two-component index $\mathbf{j}$), for example, we can adopt the following expression for a vortex whose center is at the point (1/2, 1/2):

$$\phi_{\mathbf{j}} = \Phi_{\mathbf{j}} \equiv \sum_{m=0}^{\infty} \int\!\!\int_{-\pi}^{\pi} \frac{d^2\mathbf{k}}{(2\pi)^2}\, \frac{2\pi}{\kappa(\mathbf{k})}\, (e^{-ik_y} - 1)e^{i(\mathbf{k}\mathbf{j} + k_x m)};\ \kappa(\mathbf{k}) = 4 - 2\cos k_x - 2\cos k_y\,.$$

On a trajectory corresponding to the tunneling of the core of a vortex one cell to the left we then have

$$\phi_{\mathbf{j}}(\tau)=\frac{1}{2}(\Phi_{\mathbf{j}}+\Phi_{\mathbf{j}+\mathbf{e}_x})-\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\int_{-\pi}^{\pi}\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{2\pi}{-i\omega}VG_0(\mathbf{k}\omega)(e^{-iky}-1)e^{i(\mathbf{kj}-\omega\tau)},$$

where

$$G_0(\mathbf{k},\omega)=[(\hbar^2/J)\omega^2+V\kappa(\mathbf{k})]^{-1},$$

and the excess action S_1 is

$$S_1\equiv S[\phi_{\mathbf{j}}(\tau)]-S[\Phi_{\mathbf{j}}]=\int\frac{d\omega d^2\mathbf{k}}{(2\pi)^3}\pi^2\hbar^2(V/J)G_0(\mathbf{k},\omega)\propto\hbar(V/J)^{1/2}. \quad (2)$$

The quantity S_1 determines how small the zero-temperature hopping amplitude will be: $\Delta\propto\exp(-S_1/\hbar)$. Both the dependence $S_1\propto(V/J)^{1/2}$ and the nature of the interaction of instantons (more on this below) remain the same, even if we do not change the type of potential.

In contrast with the standard problem of phase tunneling at an isolated junction with a Hamiltonian $\hat{H}_0=(J/2)\hat{n}^2-V\cos\phi$, in the problem at hand the instantons interact strongly with each other [over a long time, $\Delta S\approx\pm\int[d\omega d^2k/(2\pi)^3]2\pi^2\hbar^2(V/J)G_0(\mathbf{k},\omega)e^{-i\omega\tau}\propto 1/\tau$]. This type of instanton interaction has been seen before in an analysis of the growth of a quantum crystal.[5,6] Since the characteristics of tunneling processes are determined unambiguously by an interaction of instantons in the semiclassical approximation, we can, after reformulating the results of Ref. 6, immediately conclude that for $k_BT\gtrsim J$ the interstitial tunneling of vortices is a purely incoherent process, characterized by a tunneling probability w. In the temperature interval $J\ll k_BT\ll(JV)^{1/2}$ we have

$$w\propto\frac{\Delta^2J^{1/2}}{\hbar(k_BT)^{3/2}}\exp[2\pi(\ln 2)k_BT/J], \quad (3)$$

and at $k_BT\sim(JV)^{1/2}$ there should be a transition to an activated tunneling regime. The diffusion coefficient D is related to w by the trivial relation $D=a^2w$, where a is the lattice constant.

In the incoherent tunneling regime with which we are concerned here, incorporating the interaction of a vortex with another vortex (if we are dealing with a bound pair), with the boundaries of the sample, and with inhomogeneities reduces to multiplying (3) by a Boltzmann factor corresponding to a diffusive motion in an external potential. In the case $k_BT\ll J$, on the other hand, we need to consider both the partial restoration of a coherence in the tunneling (due to the interaction of various tunneling events) and the disruption of this coherence because of the disruption of levels in adjacent cells caused by various factors. This is an incomparably more complicated problem.

If each of the junctions is shunted by a normal resistance R, i.e., the system is described by a Euclidean action[7]

$$S=\int_{-\beta/2}^{\beta/2} d\tau\left[\sum_{j}(\hbar^2/2J)\dot\phi_j^2-\sum_{(jj')}V\cos(\phi_j-\phi_{j'})\right]$$

$$+\int\int_{-\beta/2}^{\beta/2} d\tau d\tau'\sum_{(jj')}\frac{\eta\hbar}{4\pi}\left\{\frac{\phi_j(\tau)-\phi_{j'}(\tau)-\phi_j(\tau')+\phi_{j'}(\tau')}{(\beta/\pi)\sin[(\pi/\beta)(\tau-\tau')]}\right\}^2, \qquad (4)$$

where $\eta=\hbar/4e^2R$, and $\beta=\hbar/k_BT$, the interaction of instantons at zero temperature turns out to be logarithmic, as it is for an isolated junction with dissipation.[8] Using the known properties of systems with an ohmic dissipation,[9] we find that under the condition $\eta>1/\pi$ the interstitial tunneling of vortices is incoherent down to zero temperature, and the tunneling probability w vanishes in a power-law fashion with decreasing temperature: $w\propto T^{2\pi\eta-1}$ (the exponent is calculated for a piecewise-parabolic potential).

Yet another model which we consider has the Hamiltonian

$$\hat H=\sum_{(jj')}\left[\frac{J'}{2}(\hat n_j-\hat n_{j'})^2-V\cos(\phi_j-\phi_{j'})\right]. \qquad (5)$$

In terms of a lattice of tunnel junctions, this model corresponds to the nonphysical case in which only the mutual capacitance of each pair of neighboring granules is taken into consideration. The same model applies to the free surface of a quantum crystal.[10,11] In that interpretation, the first term in (5) is the potential energy of the surface associated with the presence of steps on it, while the second describes hops of atoms along the surface. Under the conditions $J,k_BT\ll V$, the surface is in a superfluid state.[11]

For the characteristics of the instantons in model (5), we adopt those given by the same formulas as for model (1), but with $J\to J'\kappa(\mathbf{k})$. As in model (4), the interaction of the instantons is logarithmic, so that we can conclude that the tunneling probability has a power-law temperature dependence: $w\propto T^{(\pi/4)(V/J)^{1/2}-1}$. In the case of normal shunts, $2\pi\eta$ should be added to the exponent in this dependence.

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