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Phase Diagram of a Chain of Dissipative Josephson Junctions.

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Abstract. – Zero-temperature phase diagrams of a Josephson junction chain interacting capacitively with a conducting substrate are constructed both for a linear and quasi-particle Ohmic dissipation. In both cases the phase boundary between coherent (superconducting) and incoherent states of a chain as a whole is split into 4 segments by singular points. At finite temperature, zero-temperature superconductivity is destroyed and manifests itself only through algebraic decrease with temperature T of the resistance of the chain R: $R \propto T^{\lambda}$. The values of the exponent λ are found for different regions of phase diagrams, including universal values accepted by it on different segments of the phase boundary. The physical meaning of additional phase transitions present on phase diagrams is clarified.

When a conducting substrate is present a regular chain of the Josephson junctions can be described by the dimensionless Euclidean action

$$S = \sum_j \left\{ \int \mathrm{d}\tau \left[\frac{m}{2}\left(\frac{\partial \varphi_j}{\partial \tau}\right)^2 + \frac{M}{2}\left(\frac{\partial \varphi_j}{\partial \tau} - \frac{\partial \varphi_{j-1}}{\partial \tau}\right)^2 - V\cos(\varphi_j - \varphi_{j-1})\right] + S_{\mathrm{D}}[\varphi_j - \varphi_{j-1}] \right\}, \qquad (1)$$

where φ_j is the phase of the order parameter on the j-th superconducting grain, $m = (\hbar/4e^2)\,C_0$, C_0 is the capacity of each of the grains with respect to the substrate, $M = (\hbar/4e^2)\,C_1$, C_1 is the mutual capacity of the neighbouring grains $V = I_c/2e$, I_c is the critical current of a single junction. In the absence of dissipation ($S_{\mathrm{D}} \equiv 0$) the zero-temperature partition function of the model (1) is isomorphic to a partition function of a classical two-dimensional XY-model. Here instantons—extremal trajectories of the action (1) for which this or that variable $\theta_j = \varphi_j - \varphi_{j-1}$ rolls over the maximum of the cosine potential—play the role of vortices. A spatial-time distribution of the field φ far from the saddle point proves to be the same as in the vortex [1]. A logarithmic interaction of instantons can be characterized by a prelogarithmic factor $2\varkappa = 2\pi(mV)^{1/2}$, and their fugacity y is finite at $M = 0$ and decreases exponentially with increase in MV.

At $\varkappa = \varkappa_c = 2 + O(y^2)$ in an instanton gas, there occurs a phase transition from a dielectric phase into a plasma one, corresponding to a transition of a chain from a coherent (superconducting) state into an incoherent one [1, 2]. At any finite temperature T zero-temperature superconductivity is destroyed manifesting itself only through algebraic decrease of the resistance of the chain R [2] which in the semi-classical approximation can be related to the probability of the quantum-fluctuation phase-slip on each of the junctions as

$$R = 4\pi^2 \frac{\hbar}{4e^2} \frac{\hbar P}{T} \propto T^{\lambda}. \tag{2}$$

In the absence of dissipation $\lambda \approx 2\varkappa - 3$ for $\varkappa \gg 1$ and tends to I for $\varkappa \to \varkappa_c + 0$ [2].

In the present paper we shall construct a phase diagram of the model (1) and find the values of the exponent λ in its different regions for two different cases of linear and quasi-particle Ohmic dissipation.

In the case of linear Ohmic dissipation [3] we get

$$S_D[\theta] = \frac{\eta}{4\pi} \iint_{-\infty}^{\infty} d\tau\, d\tau' \left[\frac{\theta(\tau) - \theta(\tau')}{\tau - \tau'}\right]^2, \tag{3}$$

where $\eta = \hbar/4e^2 R_{sh}$ and R_{sh} is a shunting normal resistance. The presence in the action of such a nonlocal term results in the appearance of the additional logarithmic interaction (with a prelogarithmic factor equal to $2\alpha = 4\pi\eta$) for instantons located at the same site. When substituting a harmonic potential for a cosine one an overall interaction of the instantons can be described by the Green's function

$$G_0(k, \omega) = \frac{4\pi^2}{\dfrac{\omega^2}{V} + \dfrac{|\omega|}{\eta + M|\omega| + m|\omega|/2(1-\cos k)}}. \tag{4}$$

Finding by different methods (including renormgroup analysis [4]) a solution of the self-consistent equation

$$G^{-1}(k, \omega) = G_0^{-1}(k, \omega) + \Sigma_1(k, \omega);$$
$$\Sigma_1 = 2y^2 \sum_R \int_{-\infty}^{\infty} d\tau [1 - \cos(kR - \omega\tau)] \exp[-G(R=0, \tau=0) + G(R, \tau)], \tag{5}$$

describing a renormalization of the Green's function by neutral pairs of instantons, we arrive at the phase diagram depicted in fig. 1 for a limiting case of $y \to 0$ ($MV \to \infty$). In the region S_2, $\Sigma_1(k, \omega)$ does not depend on k and for small ω has the form $\Sigma_1(\omega) \approx \mu\omega^2$, corresponding to a quantitative renormalization of V: $V \Rightarrow V_R = (V^{-1} + 4\pi^2\mu)^{-1} < V$. In the region S_3, $\Sigma_1(k, \omega)$ does not depend on k either, but it has an asymptotic form $\Sigma_1(\omega) \propto \omega^{2\alpha-1}$, that results in the screening of the isotropic component of a logarithmic instanton interaction. In the region S_1, $\Sigma_1(k, \omega) \approx \mu\omega^2 + uk^2$ and a diagonal term in site number in instanton interaction proves to be screened.

All three regions S_1, S_2 and S_3 correspond to a superconducting state of a chain at zero-temperature. On the $ABCDE$ line there occurs a dissociation of instanton pairs, *i.e.* a transition of a chain into an incoherent state. In the region N, $\lim_{T\to 0} R(T) = R_{sh} > 0$. With increase in y (decrease in MV) the AB and CG lines will be shifted upwards not conserving their straightness, whereas the HC and DE lines will be lying as previously on the lines $\alpha = 1/2$ and $\alpha = 1$, respectively. The points B and C cannot merge even for $M = 0$, since they correspond to different values of V_R. A more detailed description of the procedure of phase diagram construction is relegated to a future publication [5].

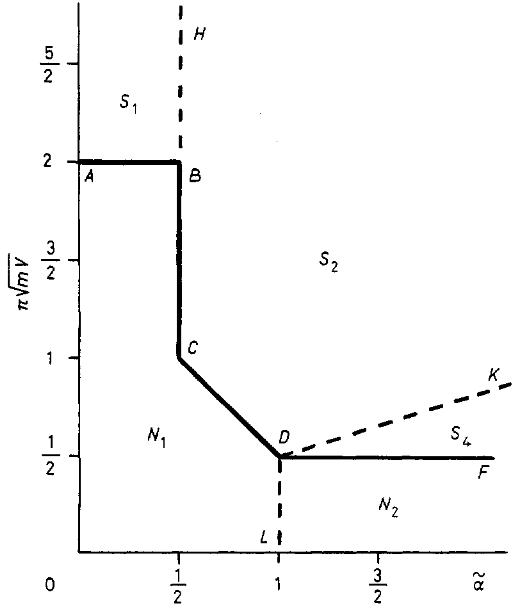

Fig. 1.

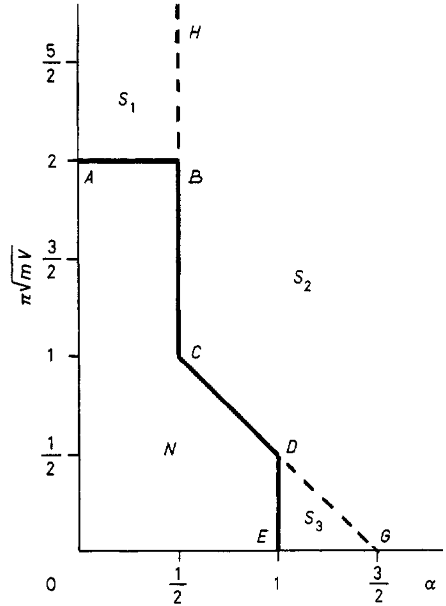

Fig. 2.

Fig. 1. – Phase diagram of a chain of Josephson junctions with linear Ohmic dissipation.

Fig. 2. – Phase diagram of a chain of Josephson junctions with quasi-particle Ohmic dissipation.

Substituting the renormalized Green's functions found by us into a general expression for the probability of the phase-slip P [2]

$$P = y^2 \sum_{R=-\infty}^{\infty} \int_{\beta/2 - i\infty}^{\beta/2 + i\infty} d\tau \exp\left[- \int_{-\pi}^{\pi} \frac{dk}{2\pi} \frac{T}{\hbar} \sum_{n=-\infty}^{\infty} [1 - \cos(kR - \omega_n \tau)] G(k, \omega_n) \right]; \tag{6}$$

$$\omega_n = \frac{2\pi T}{\hbar} n; \qquad \beta = \frac{\hbar}{T}$$

and further into eq. (3) we arrive at the following expressions for the exponent λ:

$$\lambda = \begin{cases} 2\pi (mV_{\mathrm{R}})^{1/2} - 3, & \text{in the region } S_1, & (7a) \\ 2\pi (mV_{\mathrm{R}})^{1/2} + 2\alpha - 2, & \text{in the region } S_2, & (7b) \\ 2\alpha - 2, & \text{in the region } S_3. & (7c) \end{cases}$$

For any value of MV while approaching the AB and CG lines from the side of the regions S_1 and S_2, respectively, the exponent λ will tend to a universal value $\lambda = 1$ displaying a square-root singularity. However, the region of the critical behaviour will prove to be narrower for greater MV. Within the whole region S_3 λ does not depend on m, M and V and at $\alpha \to 1 + 0$ it tends to a universal value $\lambda = 0$ (as in the case of a single junction). On the segment BC $\lim_{\alpha \to 1/2 + 0} \lambda$ is not universal and changes smoothly from 3 up to 1.

On the HB line the exponent λ has a jump with a universal value $\Delta\lambda = 2$, whereas of the DG line the value of the jump is not universal. Though on these lines in a rigorous sense the exponent λ is discontinuous, the temperature interval $0 < T \ll T_*$ of applicability of eqs. (7*a*) and (7*c*) will become more and more narrow ($T_* \to 0$) while approaching *HB* and *DG* from the left. In this case for $T \gtrsim T_*$ the dependence $R(T)$ will be described by the exponent (7*b*). Zaikin and Panukov [6] were pioneers to point out that a phase transition takes place at $\alpha = 1/2$, $mV \gg 1$. However, these authors mistook it for the transition into an incoherent state.

Now consider a chain of Josephson junctions with dissipation related to a tunnelling of single quasi-particles that can be described by the non-Gaussian nonlocal contribution to the action [7]

$$S_{\mathrm{D}}[\theta] = \frac{\tilde{\eta}}{4\pi} \iint_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{d}\tau' \left\{ \frac{4 \sin 1/4[\theta(\tau) - \theta(\tau')]}{\tau - \tau'} \right\}^2 . \tag{8}$$

In this case with respect to an on-site logarithmic interaction (characterized by a prefactor $2\tilde{\alpha} = (16/\pi)\tilde{\eta}$) instantons behave as alternating charges whatever the signs of their real charges (just as in the case of a single Josephson junction with a quasi-particle dissipation [8]). Nonetheless the upper part of the phase diagram depicted in fig. 2 for the same limiting case of $MV \gg 1$ proves to be practically the same as in the case of a linear dissipation except for the fact that at the decrease of MV the *HC* line will be shifted to the right (since the interaction of the alternating logarithmic charges is renormalizable).

The lower part of the phase diagram proves to be essentially different, since for $\tilde{\alpha} > \varkappa$ the neighbouring instantons on the same site will attract each other in the case of the identical charges as well (cf. with [9]). Such a pair can be considered as a bound complex-*a* double instanton. A long-range interaction of double instantons has only an isotropic component, thus, at $4\varkappa = 2 + O(Y^2)$, $Y \propto y^2$ (on the *DF* line in fig. 2), in the Coulomb gas of double instantons there will occur the ordinary Berezinski-Kosterlitz-Thouless transition into a plasma phase [4]. An isotropic logarithmic instanton interaction will be screened but a diagonal-in-site interaction will bind single instantons into neutral pairs or double instantons as before. With decrease of $\tilde{\alpha}$ on the *DL* line ($\tilde{\alpha} = 1 + O(y^2)$) there will be a dissociation of single instanton pairs.

Both phases N_1 and N_2 are not superconducting at zero temperature. It follows from continuity that within the whole region N_1 both single-electron tunnelling and the tunnelling of the Cooper pairs are suppressed by the Coulomb blocade, *i.e.* take place only virtually (with the obligatory quick recoil into the initial state). Thus the excited states are separated from the ground one by a gap and at $T = 0$ a chain is not conducting.

As $\tilde{\alpha}$ increases a gap in the spectrum decreases and at a strong overlap in time of virtual processes of single electron tunnelling it can vanish. It is reasonable to expect that it should be on the line of the phase transition *DL* found by us previously. Then in phase N_2 one can expect a chain to have a finite resistance at zero temperature.

In dynamics the existence of double instantons manifests itself through the possibility of 4π-phase–slips. The temperature dependence of the contribution to the chain resistance, related to these processes, can be described by the exponent

$$\lambda = 8\pi(mV_{\mathrm{R}})^{1/2} - 3 . \tag{9}$$

In the region S_4 at $T \to 0$ such processes will dominate over the usual 2π-phase–slips. While crossing the *DK* line (for $MV \to \infty$ having the form $\varkappa = (2\tilde{\alpha} + 1)/6$) the exponent λ as a function of the parameters will have a discontinuity of a derivative related to a change of the preferable channel of a phase-slip. While approaching the *DF* line, the exponent λ will tend to a unit displaying a square-root singularity as in the case of two other segments (*AB* and *CD*) of the phase boundary.

In the regions S_1 and S_2 the dependence of the exponent λ on the parameters will still be described by eqs. (7) with the substitution of $\tilde{\alpha}_R$ for α. Here, while approaching the HC line from the right λ will also have a root singularity and the value of the jump $\Delta\lambda$ on the HB line will conserve its universal value.

A semi-classical approximation used in the present paper is applicable in a wide region of the parameters, *e.g.*, at $MV \gg 1$ for arbitrary values of other parameters. In conclusion one can summarize that a transformation to an instanton gas representation and calculation of the exponent of the temperature dependence of resistance λ allowed us not only to ascertain a more subtle structure of the phase diagram of a Josephson junctions chain both with linear or quasi-particle dissipation that was made possible by using either variational approximation [10, 11] or mean-field theory [12, 13], but also to determine the universal properties of different lines of a phase diagram.

In the absence of a conducting substrate an inverse capacitance matrix $(C^{-1})_{jl}$ will have a long-range tail: $(C^{-1})_{jl} \propto |j-l|^{-1}$ $(j-l \to \infty)$ [14]. As a result an instanton interaction component not related to dissipation will diverge only as a square root from logarithm and will prove not essential as compared to a diagonal over sites logarithmic interaction caused by dissipation. In such a case the phase diagram of a chain of Josephson junctions with this or another dissipation mechanism will prove to be practically the same as for a single junction. In particular in the case of a linear Ohmic dissipation independent of the value of V a phase transition will take place at $\alpha = 1$, and exponent λ in a superconducting phase $(\alpha > 1)$ will take the value $\lambda = 2(\alpha - 1)$.

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