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# Fluctuations and Melting of the Uniaxial Vortex Crystal in a Layered Superconductor.

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**Abstract.** – The problem of vortex lattice melting is considered for a uniaxial crystal in which displacements of vortices can occur only in one plane. A flux line lattice with such properties can be obtained if magnetic field is applied to a 3D layered superconductor in parallel to the layers. Two different cases are investigated: i) that of a lattice with local elastic moduli and ii) the one explicitly incorporating the long-range vortex-vortex interaction. In both cases the phase transition is of the Berezinskii-Kosterlitz-Thouless type. For logarithmically interacting vortices we find the melting temperature to be much higher than the temperature of the transition of the layered superconductor to the normal state in the absence of the magnetic field. This means that our analysis is insufficient and that an adequate description of the flux line lattice melting should also incorporate other types of fluctuations.

Recent activity in the field of high-$T_c$ superconductivity has led to the revival of interest in the problem of vortex lattice melting. However, no physical insight beyond the Lindemann's criterion has so far been achieved. In this paper we consider a special case when the problem of vortex lattice melting can be treated analytically. This is the case of a uniaxial vortex crystal in which possible displacements of vortices can occur only in one plane.

Such vortex crystal can be formed, for example, if the magnetic field is applied to a superconductor with a well-developed layered structure in parallel to the layers (fig. 1). Then at low enough temperatures one can neglect the possibility of vortex hopping between the valleys because the energy of a kink on a vortex is large [1]. It has been conjectured by Chakravarty *et al.* [2] that flux line lattice melting in such a system takes place at low temperatures when one can really consider only uniaxial displacements of vortices. Our main result is that this assumption is not quite true and that at all temperatures at which the vortex crystal can be treated as uniaxial it remains unmelted.

We shall relate the melting transition in the vortex crystal with proliferation of topological excitations. As in the case of an ordinary crystal, relevant defects of the vortex crystal structure are dislocations. However the fact that the vortices are continuous and the

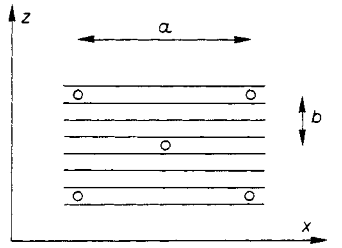


Fig. 1. – Schematic representation of the triangular flux line lattice we are studying in this paper. The magnetic field is applied along the $y$-axis which is perpendicular to the plane of the picture. $a$ and $b$ are the periods of the lattice in two directions.

crystal is uniaxial imposes severe restrictions on the orientation of Burgers' vectors (which should be parallel to the $x$-axis) and of dislocation loops (which should be parallel to the $(x, y)$-plane). In terms of topological defects the melting transition can be described as the appearance of thermodynamically stable infinite dislocations, whereas in the low temperature (ordered) phase dislocations will be present only as closed loops. When the vortex crystal is melted, the planes where the vortices are localized will divide the superconductor into layers between which there will be no phase coherence. The possibility of a transition of a superconductor to such a state due to the application of a magnetic field along the layers has been suggested by Efetov [3].

Although in the second half of the paper we shall use a more sophisticated approach, to begin with, we study a simplest model of a uniaxial vortex crystal. We shall assume that long-wavelength fluctuations of such a crystal can be described by three local moduli: a stress modulus $\lambda_x$, a tilt modulus $\lambda_y$ and a shear modulus $\lambda_z$:

$$H = \frac{1}{2}\int d^3 \boldsymbol{r}\left[\lambda_x\left(\frac{\partial u}{\partial x}\right)^2 + \lambda_y\left(\frac{\partial u}{\partial y}\right)^2 + \lambda_z\left(\frac{\partial u}{\partial z}\right)^2\right] \equiv \frac{1}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\Lambda(\boldsymbol{k})\, u(\boldsymbol{k})\, u^*(\boldsymbol{k}) \tag{1}$$

with

$$\Lambda(\boldsymbol{k}) = \lambda_x k_x^2 + \lambda_y k_y^2 + \lambda_z k_z^2 . \tag{2}$$

It is convenient for us to consider $u$ and $x$ expressed in units of $a$ (lattice constant in the $x$-direction) and $z$ in units of $b$ (lattice constant in the $z$-direction). The long-range interaction of dislocations in such a crystal is then described by the Green's function

$$U_{\alpha\beta} = \frac{\lambda_x \lambda_y \lambda_z}{\Lambda(\boldsymbol{k})\,\lambda_\alpha}\,\delta_{\alpha\beta} . \tag{3}$$

One can easily take dislocations into account, substituting the last term in eq. (1) by the periodic term

$$\widetilde{H} = \frac{1}{2}\sum_{x,z}\int dy\left\{\lambda_x(\nabla_x u)^2 + \lambda_y\left(\frac{\partial u}{\partial y}\right)^2 + \tilde{\lambda}_z[1-\cos(2\pi\nabla_z u)]\right\}, \tag{4}$$

where $\nabla_x$ and $\nabla_z$ stand for the lattice differences. For simplicity here we treat the lattice as square. Both orientation and interaction of the topological excitations (vortex loops) of the Hamiltonian (4) will be the same as for dislocation loops in our vortex crystal. This will permit us to consider the Hamiltonian (4) as an approximate description of this crystal.

Expanding the partition function

$$Z = \int D\{u\} \exp[-\widetilde{H}\{u\}/T], \tag{5}$$

in powers of $(\tilde{\lambda}_z/T)$ one obtains the partition function of a layered Coulomb gas [4]. The interaction $g(\boldsymbol{r})$ of charges in this gas diverges logarithmically not only for charges in the same layer, but also for charges in the neighbouring layers. The Fourier transform of $g(\boldsymbol{r})$ has the form

$$g(\boldsymbol{k}) = 4\pi^2 T 2(1-\cos k_z)/\Lambda_{\|}(\boldsymbol{k}), \qquad \Lambda_{\|}(\boldsymbol{k}) = \lambda_x 2(1-\cos k_x) + \lambda_y k_y^2. \tag{6}$$

The fugacity of the charges proves to be equal to $\tilde{\lambda}_z/2T$.

The function $\Lambda_{\|}(\boldsymbol{k})$ which appears in eq. (6) is the propagator describing the harmonic part of the Hamiltonian (4). The long-range interaction of our layered Coulomb gas is determined by the behaviour of $\Lambda_{\|}(\boldsymbol{k})$ at small $k$.

According to ref. [4] the phase transition in such a system can be described by the well-known Kosterlitz renormalization equations [5]. The transition takes place when the renormalized prefactor in the logarithmic interaction of the charges becomes equal to 4. For $\tilde{\lambda}_z \ll \lambda_x$ this corresponds to

$$T_{\rm m} = T_{\rm m}^0 + O(\tilde{\lambda}_z^2/T_{\rm m}), \qquad T_{\rm m}^0 = \frac{1}{\pi}(\lambda_x \lambda_y)^{1/2}. \tag{7}$$

Thus the value of $T_{\rm m}$ for $\lambda_z \to 0$ proves to be not dependent on the shear modulus $\lambda_z$. In ref. [4] this very phase transition has been analysed in more detail.

In terms of the renormalization equations $T_{\rm m}^0$ is the temperature at which the operator $\cos(2\pi\nabla_z u)$ becomes marginal. In a more realistic description one should take into account that the relative displacements of vortices in neighbouring layers can be large and consider something like

$$\cos 2\pi\{u(\boldsymbol{r}+\boldsymbol{e}_z+\boldsymbol{e}_x\nabla_z u) - u(\boldsymbol{r})\}$$

instead of $\cos(2\pi\nabla_z u)$, but this will not change the value of $T_{\rm m}^0$.

It is worth mentioning that the application of Lindemann's criterion in the form $\langle(\nabla_z u)^2\rangle \sim$ const would lead to $T_{\rm m} \sim T_{\rm m}^0/\ln(T_{\rm m}^0/\tilde{\lambda}_z)$. Thus in our problem Lindemann's criterion turns out to be of no avail. It is so because the quasi-long-range order in each layer of vortices still persists when the relative displacements of vortices in neighbouring layers are large compared with the lattice constant and the scaling properties of $\tilde{\lambda}_z$ are related to this internal periodicity of each layer.

Up to now for methodical reasons we have treated a vortex crystal as having local elastic moduli. This can be done only if the vortex-vortex interaction is sufficiently short range. Dealing with flux line lattice in type-II superconductor one would be naturally more interested in the case of a relatively dense lattice (with a strong interaction between vortices) when there are more grounds to disregard the influence of pinning. The long-range interaction of the vortices leads to a nonlocality of the elastic energy which should be explicitly taken into account. We do this in what follows.

Long-wavelength fluctuations of a layered superconductor in the London approximation can be described by the Hamiltonian

$$H = \int {\rm d}^3\boldsymbol{r}\left[\sum_\alpha \frac{J_\alpha}{2}\left(\nabla_\alpha\phi - \frac{2e}{c}A_\alpha\right)^2 + \frac{1}{8\pi}({\rm rot}\,\boldsymbol{A})^2\right], \qquad J_x = J_y = J_{\|} \equiv J_{\|}^0/d, \tag{8}$$

where $J_{\parallel}^{0}$ stands for the superfluid density of each layer and $d$- for the spacing between layers. Then the interaction of the vortices that are arbitrarily curved in the $(x, z)$-plane will be given by the Greenn's function [4]:

$$G(\boldsymbol{k}) = \frac{4\pi^2}{D^{-1} + J_z^{-1} k_{\parallel}^2 + J_{\parallel}^{-1} k_z^2}, \quad D^{-1} = 4\pi(2e/c)^2, \quad k_{\parallel}^2 = k_x^2 + k_y^2, \tag{9}$$

where we consider the distances between the vortices to be much larger than the coherence lengths in the respective directions. For distances smaller than the penetration length this interaction has the form of current-current interaction.

After performing standard calculations [6] one obtains that in this case the energy of fluctuations of a regular triangular vortex crystal (see fig. 1) shall be described by the propagator

$$\Lambda(\boldsymbol{k}) = \Lambda^{\mathrm{l}}(\boldsymbol{k}) + \Lambda^{\mathrm{n}}(\boldsymbol{k}) \tag{10}$$

incorporating an almost local term

$$\Lambda^{\mathrm{l}}(\boldsymbol{k}) \approx \lambda_{\parallel}^{\mathrm{l}} k_{\parallel}^2 + \lambda_z k_z^2 \tag{11}$$

and a substantially nonlocal term $\Lambda^{\mathrm{n}}(\boldsymbol{k})$. For $k_{\parallel}^2 \ll a^{-2}$

$$\Lambda^{\mathrm{n}}(\boldsymbol{k}) \approx \frac{4\pi k_{\parallel}^2}{D^{-1} + J_z^{-1} k_{\parallel}^2 + J_{\parallel}^{-1}[2(1-\cos bk_z)/b^2]}. \tag{12}$$

In the following we shall be mostly interested in the case when the periods of the vortex lattice are larger than the corresponding penetration lengths, that is

$$a^2 \gg D/J_z; \qquad b^2 \gg D/J_{\parallel} \tag{13}$$

when the term $D^{-1}$ in the denominator of eq. (12) can be omitted.

The coefficients $\lambda_{\parallel}^{\mathrm{l}}$ and $\lambda_z$ in eq. (11) depend on the parameter $\kappa = J_{\parallel} b^2 / J_z a^2$ describing the configuration of the lattice. The value of $\kappa$ is determined both by the method of preparing the flux line lattice and by the value of the external magnetic field. The equilibrium lattice corresponds to $\kappa = \kappa_0 = 3/4$. For $\kappa > \kappa_0$ the lattice is supercompressed in the $x$-direction. Had it not been for the uniaxiality of the vortex motion it would have relaxed to the state with a smaller value of $b$ and a larger value of $a$. With decreasing $\kappa$ down to $\kappa_c \sim 1/3$ there will be a phase transition related to deformation of the lattice cell [1]. The value of $\kappa$ can be changed continuously by decreasing the magnetic field. The limit of high fields corresponds to $b = d$ and $\kappa \gg 1$.

The parameter $\lambda_z$ strongly depends on $\kappa$. For $\kappa \gg 1$ it is exponentially small:

$$\lambda_z \approx 8\pi^3 (J_{\parallel} J_z)^{1/2} (b^3/a) \exp[-2\pi\kappa^{1/2}], \tag{14}$$

whereas for $\kappa \sim 1$ this smallness disappears. On the contrary the dependence of $\lambda_{\parallel}^{\mathrm{l}}$ on $\kappa$ is very weak. Both for $\kappa \gg 1$ and for $\kappa \sim \kappa_0$ one can take

$$\lambda_{\parallel}^{\mathrm{l}} \approx \pi (J_{\parallel} J_z)^{1/2} ab \ln(a/\xi). \tag{15}$$

Here $\xi$ stands for the coherence length in the $(x, z)$-plane.

Thus we have found the form of the nonlocal propagator (10) describing fluctuations of the vortex crystal in the harmonic approximation (see eqs. (11)-(15)). Then after the substitution of the shear term in the Hamiltonian by the periodic one

$$\frac{\lambda_z}{2}\left(\frac{\mathrm{d}u}{\mathrm{d}z}\right)^2 \Rightarrow \frac{\tilde{\lambda}_z}{2}\left\{1-\cos\frac{2\pi}{a}[u(\boldsymbol{R})-u(\boldsymbol{R}')]\right\}, \tag{16}$$

where $\boldsymbol{R}$ and $\boldsymbol{R}'$ are the neighbouring sites in the neighbouring layers of the lattice, one can apply the same procedure of transformation to the layered Coulomb gas. The interaction of the charges will be determined by the remaining part of the propagator

$$\Lambda_{\parallel}(\boldsymbol{k}) = \Lambda(\boldsymbol{k}) - \lambda_z k_z^2 \approx \Lambda^{\mathrm{n}}(\boldsymbol{k}) + \lambda_{\parallel}^{1} k_{\parallel}^2 \tag{17}$$

and in the long-wavelength limit will have the form

$$g_{\parallel}(\boldsymbol{k}) \approx \frac{(2\pi b)^2 T}{\Lambda_{\parallel}(\boldsymbol{k})} 2(1-\cos bk_z). \tag{18}$$

This change of the form of the propagator does not affect properties of the transition which remains of the Berezinskii-Kosterlitz-Thouless type. As for the previously considered model it will take place when the prelogarithmic factor in the intralayer interaction of charges is equal to 4. For

$$\kappa \gg \kappa_* = [3\ln(a/\xi)/4\pi]^2, \tag{19}$$

that is for almost the entire range of the parameters the second term in eq. (10) will be dominant, giving

$$T_{\mathrm{m}} = [1+O(Y^2)]\,T_{\mathrm{m}}^0; \qquad T_{\mathrm{m}}^0 = \frac{4\pi}{3} J_{\parallel} b; \qquad Y \propto \exp[-\kappa^{1/2}], \tag{20}$$

whereas for $\kappa \ll \kappa_*$ the value of $T_{\mathrm{m}}^0 \approx (J_{\parallel} J_{\perp})^{1/2} a \ln(a/\xi)$ is even larger.

Thus we have found that the melting temperature of the relatively dense flux line lattice $T_{\mathrm{m}}$ only weakly (for $\kappa \gg 1$ exponentially weakly) depends on $J_z$ and $a$. Expressing $J_{\parallel}$ via layers' characteristics one obtains

$$T_{\mathrm{m}} \approx (4\pi/3) J_{\parallel}^0 (b/d) \sim 3(b/d)\, T_{\mathrm{c}}, \tag{21}$$

where $T_{\mathrm{c}} \sim (\pi/2) J_{\parallel}^0$ is the temperature of the phase transition in the layered system in the absence of the magnetic field.

Unfortunately for all possible values of $b$, including the smallest $b = d$ eq. (21) shows that $T_{\mathrm{m}}$ is much higher than $T_{\mathrm{c}}$. This means that all expectations to be able to describe the melting of vortex crystal in a layered superconductor in terms of uniaxial fluctuations of vortices with pairwise interaction are not justified. Everywhere in the parameter domain where the approximation used in the second half of this paper is applicable the vortex crystal remains unmelted. Melting will thus occur in a closer vicinity of $T_{\mathrm{c}}$, where one should take into account some other types of fluctuations, possibly including, i) appearance of thermally activated vortex loops, ii) fluctuations of the order parameter modulus and iii) hopping of vortices to neighbouring valleys, which have not been studied in this paper.

The method can be easily extended to other crystals, formed by any kind of lines, with any law of the line-line interaction.

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For the case of noninteracting vortices the same problem was studied by Kolomejsky and Mikheev [7]. The author is grateful to L. MIKHEEV for useful discussions.

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