Observation of Domain-Wall Superlattice States in a Frustrated Triangular Array of Josephson Junctions

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(Received 25 August 1993)

A family of ground states at frustrations $f=\frac{1}{2}-\frac{1}{2}N^{-1}$, with N an integer ≥ 2, has been observed in the superfluid response of a triangular Josephson junction array as a function of the magnetic flux ($\propto f$) threading a unit cell. These states can be constructed by introducing one- or two-dimensional superlattices of domain walls on the background of the checkerboard ground state at $f=\frac{1}{2}$, the 1D superlattice (striped phase) being energetically more favorable for $N>3$. A structural change of the ground state from the striped phase to a 2D superlattice of vacancies is predicted at $f_c \approx 0.468$.

PACS numbers: 74.50.+r, 75.10.Hk

The concept of frustration, first introduced by Toulouse [1], plays a fundamental role in modern condensed-matter physics and statistical mechanics. It is central, for instance, in the theoretical understanding of spin glasses, quasicrystals, and incommensurate systems. In this context, planar arrays of Josephson junctions exposed to a perpendicular magnetic field provide a unique system where the influence of a tunable level of frustration can be studied in a variety of topologies ranging from periodic to random structures, including quasiperiodic and fractal lattices. Such systems are a physical realization of the frustrated classical XY model where the degree of frustration is governed by a parameter f expressing the magnetic flux threading an elementary cell of the array in units of the superconducting flux quantum.

The determination of the ground state of a Josephson junction (JJ) array at arbitrary frustration is a delicate problem which has been addressed by several authors. In their pioneering work, Teitel and Jayaprakash [2], considering only rational f ($f=p/q$, p and q coprime integers) and assuming that the corresponding commensurate ground states of the vortex lattice are periodic with a $(q\times q)$ unit cell, were the first to determine the structure of a few low-order (small q) vortex configurations in a square JJ array. Relying on the same assumption, Halsey [3] subsequently conjectured that, for a selected class of frustrations (f rational and quadratic irrational) in the range $\frac{1}{3}\leq f\leq \frac{1}{2}$, the ground state of a square array has a quasi-one-dimensional "staircase" structure resulting in a "striped" phase characterized by a sequence of parallel *domain walls* separating $f=\frac{1}{2}$ "checkerboard" regions of opposite chirality. More recently, however, numerical calculations [4] revealed that, at large q, ground states close to $f=\frac{1}{2}$ no longer exhibit the $(q\times q)$ periodicity, thereby disproving, in part, Halsey's conjecture.

In this Letter, we focus on frustrated *triangular* JJ arrays and report the discovery of a new set of commensurate states which unambiguously demonstrates the central role of domain walls in determining the system's ground-state configurations. Besides their intrinsic relevance for the understanding of frustration, vortex dynamics, and critical phenomena in JJ arrays, our results should further stimulate the growing interest in studies probing the microscopic structure of magnetic vortex phases using novel imaging techniques [5].

Compared to other lattice structures, the potential-energy barrier opposing vortex motion in triangular JJ arrays is so low [6] that their superfluid (or diamagnetic) response $S(f)$ to a small low-frequency electromagnetic excitation is extremely sensitive to the fine tuning of f near a commensurate state. This distinctive feature results in an exceptionally rich fine structure [7] providing a unique laboratory to identify specific families of ground states. In the following we report measurements of $S(f)$ in which a particular sequence of quantum structures corresponding to commensurate states defined by $f=\frac{1}{2}-\frac{1}{2}N^{-1}$, where N is an integer such that $N\geq 2$, was observed. By a suitable choice of temperature, we were able to resolve structures up to $N=8$. Relying on the observation that these structures appear at frustrations such that their deviation $\Delta f\equiv \frac{1}{2}-f=\frac{1}{2}N^{-1}$ from $f=\frac{1}{2}$ is *linear* in N^{-1}, we show that the corresponding states can be deduced from the checkerboard state at full frustration ($f=\frac{1}{2}$) by introducing *linear* defects creating either a one-dimensional (1D) superlattice of parallel domain walls (striped phase) or a two-dimensional (2D) periodic network of intersecting domain walls (hexagonal phase) separating $f=\frac{1}{2}$ regions of opposite chirality. Theoretical considerations reveal that the striped phase has the lowest energy for $N>3$. Moreover, by comparing the energy of the striped phase to that of a competing 2D triangular superlattice of *pointlike* defects (vacancies) commensurate with the triangular vortex lattice at $f=\frac{1}{2}$, we find that the 2D vacancy superlattice becomes energetically more favorable for $f>f_c\approx 0.468$. However, being beyond experimental resolution, the structural crossover from the striped-phase to the vacancy-superlattice states was not observed.

The JJ array studied in this work is the same of the investigations of Ref. [7]. It consists of $\sim 10^6$ proximity-

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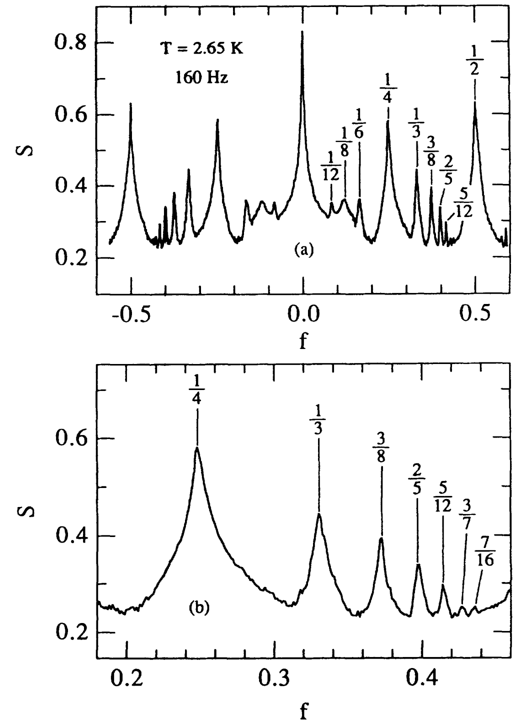


FIG. 1. (a) Normalized superfluid response of a triangular Josephson junction array as a function of frustration at 160 Hz; (b) expanded view of the interval $\frac{1}{4} < f < \frac{1}{2}$.

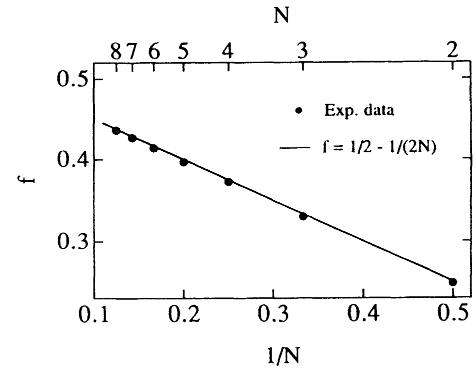


FIG. 2. Dependence of the peak positions of Fig. 1(b) on the inverse sequential integer N^{-1} and comparison with the relation $f = \frac{1}{2} - 1/2N$.

effect coupled Pb/Cu/Pb junctions sitting midway on the sides of a triangular lattice with a lattice spacing a of 15 μm. The superfluid response $S(f)$ was measured at 160 Hz with a SQUID-operated ac bridge detecting the mutual inductance change of a drive-receive coil system [8] induced by the supercurrents flowing in the JJ array in response to a weak ac field (~ 1 nT rms at the center of the array).

In Fig. 1 the superfluid response (normalized to that of a perfectly diamagnetic sample) is shown as a function of frustration at $T = 2.65$ K or, more significantly, at $\tau = 0.09$, where τ is the reduced temperature [6] relevant for the statistical mechanics of the system. At this temperature, superconducting phase coherence is expected to survive in a large number of commensurate states [9], but vortex-lattice defects created in their neighborhood by excess or missing vortices are still mobile enough [7] to sharpen the fine structure substantially, thereby enhancing resolution. Besides the central prominent peak at $f = 0$ and weaker structures at $f = 1/q$ with $q = 6$, 8, and 12, $S(f)$ exhibits a characteristic sequence of sharp peaks at $f = \frac{1}{4}, \frac{1}{3}, \frac{3}{8}, \frac{2}{5}, \frac{5}{12}$ whose strength decreases with increasing q [Fig. 1(a)]. Expansion of the interval $\frac{1}{4} < f < \frac{1}{2}$ [Fig. 1(b)] reveals two additional peaks of the same series at $f = \frac{3}{7}$ and $f = \frac{7}{16}$. As shown in Fig. 2, the positions of these commensurate structures are very accurately fitted by the expression $f = \frac{1}{2} - \frac{1}{2} N^{-1}$ with $N = 2, 3, \ldots, 7, 8, \ldots$ showing that the sequence obeys the relation $\Delta f = \frac{1}{2} N^{-1}$.

In order to elucidate the nature of the commensurate states at $f = \frac{1}{2} - \frac{1}{2} N^{-1}$, we first recall that the ground state at $f = \frac{1}{2}$ has a checkerboardlike structure consisting of elementary triangular cells with alternating positive and negative vorticities. In a Coulomb gas representation [3,10], such a state can be conceived of as a lattice of negative (positive) integer vortex charges on a positive (negative) background. To ensure global charge neutrality, for $f < \frac{1}{2}$ charged defects must appear on the background of the regular checkerboard state in order to make the actual vortex density less than that at $f = \frac{1}{2}$. Since the structure of the ground state at $f = \frac{1}{2}$ allows for the creation of both pointlike (vacancies) and linear (domain walls) defects, there are three main possibilities to construct ground states in the vicinity of $f = \frac{1}{2}$: (i) a 2D lattice of vacancies, (ii) a 1D sequence of parallel domain walls, and (iii) a 2D network of crossing (or possibly ramifying) domain walls of different orientations. Also, the possibility of ground states in which both pointlike and linear defects are present [11] cannot be excluded *a priori*.

A ground state involving some regular arrangement of defects is expected to be more stable against thermal fluctuations when it is *commensurate* with the underlying lattice. This occurs if $\mathbf{g} = \mathbf{q}$, where $\mathbf{g}$ is a vector of the reciprocal defect lattice and $\mathbf{q}$ a wave vector describing the modulation provided by the (triangular) substrate. In particular, an equilateral triangular superlattice of vacancies is commensurate with the triangular lattice of vortices in the $f = \frac{1}{2}$ ground state [Fig. 3(c)] if $\Delta f = [2(L^2 + M^2 - LM)]^{-1}$, where L and M are integers. For all other (intermediate) values of f the periodic 2D force field created by the underlying lattice will deform the "natural" triangular superlattice of vacancies, thereby making it more vulnerable to thermal fluctuations.

Within the family of vacancy-superlattice states, the most symmetric ones, corresponding to orientations such that $L = M$ and $L = -M$, are expected to be the most

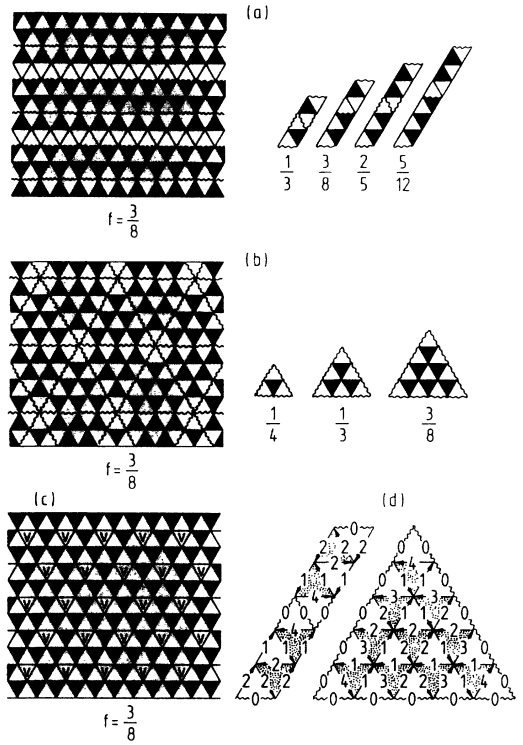


FIG. 3. Structure of (a) the striped phase, (b) the hexagonal phase, and (c) the vacancy (V) superlattice phase for $f=\frac{3}{8}$. In (a) and (b) a few superlattice elementary cells at other $f=\frac{1}{2}-1/2N$ are also shown. Wavy lines denote domain walls. (d) Distribution of the gauge-invariant phase differences (in units of $\pi/5$) at $f=\frac{2}{5}$ in the elementary cells of the striped and hexagonal phases.

stable ones. Structures in $S(f)$ arising from such states should therefore occur at frustrations such that $\Delta f \propto M^{-2}$, in striking contrast with the linear dependence of Δf on N^{-1} emerging from the data of Fig. 2. This strongly supports the idea that the sequence of peaks observed in $S(f)$ corresponds to a set of ground states which stem from the $f=\frac{1}{2}$ state by introducing a system of linear defects (domain walls) commensurate with the underlying lattice.

We have constructed two sets of states, one based on a 1D sequence of parallel domain walls [striped phase, Fig. 3(a)] and the other on a 2D triangular network of intersecting domain walls [hexagonal phase, Fig. 3(b)], and discovered that both families correspond to the series of peaks at $f=\frac{1}{2}-\frac{1}{2}N^{-1}$ observed in $S(f)$. As it is impossible to discriminate between the two configurations on the basis of the experimental data, it is necessary to compare their energies. By imposing supercurrent conservation at the nodes and fluxoid quantization in the triangular loops of the array, the distribution of the gauge-invariant phase differences $\{\theta_{ij}\}$ on the bonds $\langle ij\rangle$ of the lattice can be found explicitly for both families of states. As shown in Fig. 3(d) for $f=\frac{2}{5}$, this results in a very regular pattern exhibiting simple recursive properties. Summation of the bond contributions $\{-J\cos\theta_{ij}\}$, where J is the Josephson coupling energy, shows (see Table I) that the energy (per site) E_S of the striped phase

$$E_S=-\frac{J}{N}\sum_{m=0}^{N-1}\left[2\cos\frac{\pi(N-1-2m)}{2N}+\cos\frac{2\pi m}{N}\right]=-\frac{2J}{N\sin\pi/2N} \tag{1}$$

is always lower, if $N>3$, than the energy E_H of the hexagonal phase:

$$E_H=-\frac{3J}{N^2}\sum_{m=1-N}^{N-1}(N-|m|)\cos\frac{\pi m}{N}=-\frac{6J}{N^2(1-\cos\pi/N)}\,. \tag{2}$$

For $N=3$ ($f=\frac{1}{3}$) Eqs. (1) and (2) predict $E_S=E_H=-4J/3$. However, as shown by free-energy calculations [12], at nonzero temperatures spin-wave-like excitations of the phase system remove the degeneracy favoring the hexagonal phase for a strictly sinusoidal current-phase relation. Similar considerations show that, among the various regular domain-wall structures [10] which correspond to the ground state at $f=\frac{1}{4}$ ($N=2$), the hexagonal phase is the most stable against thermal fluctuations [12].

It is important to notice that in the limit $N\to\infty (f\to\frac{1}{2})$ both E_S and E_H are larger than $E_0=-3J/2$, the energy of the ground state at $f=\frac{1}{2}$ [9], implying that the energy of the linear defects does not vanish at arbitrarily small defect concentrations, i.e., for $\Delta f\to 0$. This is a manifestation of the long-range Coulomb interaction between pointlike defects in two dimensions and of the fact that linear defects can be

TABLE I. Comparison of the energies (per site and in units of J) of the striped phase (E_S), the hexagonal phase (E_H), and the vacancy-superlattice phase (E_V) at frustration $f=\frac{1}{2}-\frac{1}{2}N^{-1}$.

N	E_S	E_H	E_V
2		−1.5000	
3	−1.3333	−1.3333	−1.1133
4	−1.3066	−1.2803	−1.0607
5	−1.2944	−1.2567	
6	−1.2879	−1.2440	
7	−1.2840	−1.2365	−1.1453
8	−1.2815	−1.2316	
9	−1.2797	−1.2283	−1.1869
...			
12	−1.2769	−1.2228	−1.2341
...			
16	−1.2753	−1.2198	−1.2783
...			
∞	−1.2732	−1.2159	−1.5000

thought of as a superposition of pointlike defects [11]. By adding up the logarithmic potentials of pointlike defects to a "Coulomb potential" of a linear defect varying as $|x|$ in the direction x perpendicular to the wall and taking into account the compensating screening effect provided by the background, the total energy density of an infinite sequence of equally charged domain walls turns out to be finite and independent of the wall concentration.

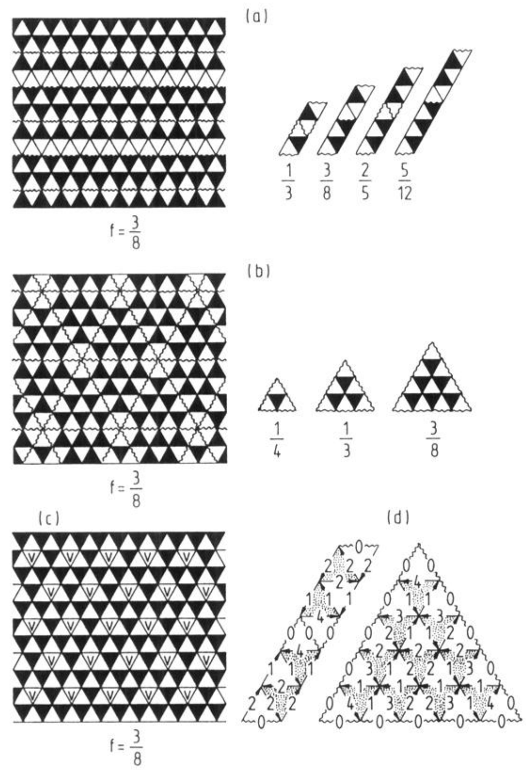
On the other hand, calculations of the energy E_V of the vacancy superlattice show that E_V should approach E_0 with decreasing vacancy concentration. Thus, for frustrations sufficiently close to $f=\frac{1}{2}$ the true ground state should have the structure of a vacancy superlattice [Fig. 3(c)]. Unlike E_S and E_H, however, no general analytic expression can be derived for E_V, which was computed from the $\{\theta_{ij}\}$ of a few vacancy-superlattice states satisfying both $\Delta f=[2(L^2+M^2-LM)]^{-1}$ and $\Delta f=\frac{1}{2}N^{-1}$. As shown in Table I, where E_S, E_H, and E_V are compared, the striped phase has the lowest energy in the interval $\frac{1}{3}<f<f_c$ with $f_c\approx 0.468$, whereas for $f_c<f<\frac{1}{2}$ the vacancy superlattice is the most favorable configuration. Notice that, in this respect, the situation is similar to that found for square JJ arrays where, however, $f_c\approx 0.438$ [4]. Since structures corresponding to commensurate states near f_c are too weak to be resolved in our $S(f)$ measurements, the crossover from the striped-phase sequence ($\Delta f\propto N^{-1}$) to the vacancy-superlattice series ($\Delta f\propto M^{-2}$) was not observed.

Before concluding, we would like to stress that the calculations of E_S, E_H, and E_V were performed for an array with infinite penetration depth $\lambda_\perp$ [6]. For finite $\lambda_\perp$, cell self-inductance and mutual inductance coupling between cells [13] will affect the energy calculations. However, since $\lambda_\perp/a\approx 30$ at the temperature of interest ($T=2.65$ K), corrections are expected to be small and the overall picture emerging from our study will not be significantly modified. This conclusion is corroborated by the observation that the sequence at $f=\frac{1}{2}-\frac{1}{2}N^{-1}$, although less rich ($N<8$), persists at higher temperatures where $\lambda_\perp$ becomes comparable to the sample size and finite-$\lambda_\perp$ corrections are irrelevant.

In conclusion, high-resolution measurements of the superfluid response of a triangular Josephson junction array exposed to a magnetic field have provided novel insight into the nature of the ground state of a frustrated system. We have shown that a particular family of commensurate states observed at frustrations f such that $f=\frac{1}{2}-\frac{1}{2}N^{-1}$, where N is an integer ≥ 2, can be constructed from the checkerboard state at $f=\frac{1}{2}$ by introducing either a one-dimensional superlattice of parallel domain walls (striped phase) or a two-dimensional triangular superlattice of intersecting domain walls (hexagonal phase). The striped phase was found to have the lowest energy for $N>3$. Although not observed in our experiment, the ground state is predicted to undergo a structural transition from the striped phase to a two-dimensional superlattice of vacancies at a critical frustration $f_c\approx 0.468$.

One of us (P.M.) would like to thank S. Teitel for a clarifying discussion on domain walls. This work was supported by the Swiss National Science Foundation.

[1] G. Toulouse, Commun. Phys. **2**, 115 (1977).
[2] S. Teitel and C. Jayaprakash, Phys. Rev. Lett. **51**, 1999 (1983).
[3] T. C. Halsey, Phys. Rev. B **31**, 5728 (1985); Physica (Amsterdam) **152B**, 22 (1988).
[4] M. R. Kolahchi and J. P. Straley, Phys. Rev. B **43**, 7651 (1991); J. P. Straley and G. M. Barnett, Phys. Rev. B **48**, 3309 (1993).
[5] H. D. Hallen *et al.*, Phys. Rev. Lett. **71**, 3007 (1993); L. N. Vu *et al.*, Appl. Phys. Lett. **63**, 1693 (1993).
[6] C. J. Lobb *et al.*, Phys. Rev. B **27**, 150 (1983).
[7] R. Théron *et al.*, Phys. Rev. Lett. **71**, 1246 (1993).
[8] B. Jeanneret *et al.*, Appl. Phys. Lett. **55**, 2336 (1989).
[9] W. Y. Shih and D. Stroud, Phys. Rev. B **30**, 6774 (1984).
[10] S. E. Korshunov, J. Stat. Phys. **43**, 17 (1986).
[11] S. Teitel, Physica (Amsterdam) **152B**, 30 (1988).
[12] A. Vallat, S. E. Korshunov, and H. Beck (private communication).
[13] J. R. Phillips *et al.*, Phys. Rev. B **47**, 5219 (1993).



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