Physica B 194–196 (1994) 1725–1726
North-Holland

PHYSICA B

Observation of dilational symmetry breaking in a superconducting array of Sierpinski gaskets

R. Meyer, B. Jeanneret, S. E. Korshunov* and P. Martinoli

Institut de Physique, Université de Neuchâtel, 2000 Neuchâtel, Switzerland
* L.D. Landau Institute for Theoretical Physics, Kosygina 2, 117334 Moscow, Russia

The inverse sheet kinetic inductance L_k^{-1} of a periodic array of n^{th} order Sierpinski gaskets has been measured as a function of frustration f, f being the number of magnetic flux quanta in the elementary triangular cell of the fractal structure . The Josephson junction array shows prominent oscillations only for frustrations corresponding to multiples of $f_n=1/(2\times4^n)$. A simple model taking into account the interplay between fractal and two-dimensional (2D) régime has been developed to calculate L_k^{-1} for the gasket array. It is shown that the periodic boundary conditions imposed by the 2D lattice are responsible for the observed oscillations.

1. Introduction

Since the beginning of the high-T_c era, disorder in superconductors is an attractive and important subject. Geometrical disorder has been elegantly described by percolating models [1], where the concept of self-similarity plays a key role. Therefore, a thorough study of the basic physics of fractals is a useful step towards the understanding of disorder in superconductors.

In this paper, we study the ac linear response of a periodic array of Sierpinski gaskets (SG) exposed to a weak magnetic field H. The array is made of SNS (Pb/Cu/Pb) Josephson junctions. Similar systems have been recently investigated [2] to demonstrate the crossover from the fractal to the 2D régime as observed in the low-field phase boundary $T_c(H)$. In this paper we show that the interplay between the fractal and the 2D régime has a profound impact on the magnetic properties of the system below $T_c(H)$. A simple model, where the periodic boundary conditions imposed by the 2D lattice are essential, accounts for our observations.

2. Experimental results

The array structure is composed of interconnected 2nd order SG. A unit cell is shown in Fig.1. The zero-field superconducting-to-normal transition temperature is $T_c(0) = 2.8$ K.

The sample ac response at T = 2 K was measured as a function of the frustration f using our standard two-coil technique [3] (f defines the number of flux quanta in an elementary triangular cell of the gasket). The inverse sheet kinetic inductance L_k^{-1} was then extracted using a numerical inversion procedure [3]. The result is shown in Fig.1 for an excitation frequency of 5 kHz. In striking contrast to the much smoother response of a single SG [4], $L_k^{-1}(f)$ of the periodic lattice of SG exhibits sharp oscillations. In the next section, we develop a simple model explaining these different behaviors.

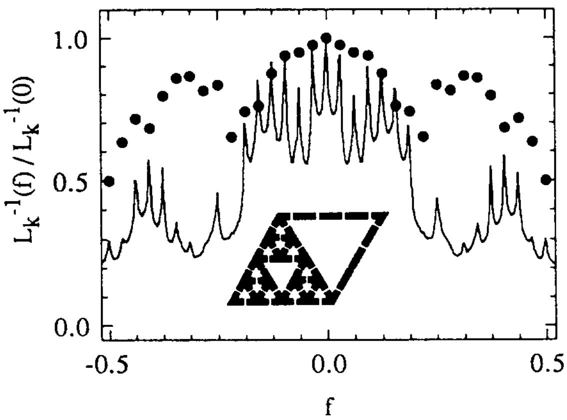

Fig.1: L_k^{-1} of the array of SG as a function of f. The experimental data (line) are compared to the model of Sec.3 (dots). The inset shows a unit cell of the array.

3. Theoretical model

The gauge invariant phase differences $\{\gamma_{ij}\}$ across the junctions are determined by current conservation at each node j, $\Sigma_i \sin\gamma_{ij} = 0$ and fluxoid quantization in the loops of various sizes formed by the array, $\Sigma_{loop}\, \gamma_{ij} = 2\pi\,(k_h - 4^h f)$. The index h labels the different families of loops (h=0,...,n-1) and k_h is the number of vortices in a loop of species h. For each loop and a given f, k_h has to be chosen in order to minimize the array's energy $E \propto -\Sigma_{array} \cos\gamma_{ij}$. The

0921-4526/94/$07.00 © 1994 - Elsevier Science B.V. All rights reserved
SSDI 0921-4526(93)1444-Q

integers $\{k_h\}$ for the ground state configurations correspond to highly symmetric vortex distributions [4,5]. Consequently, for a single n^{th} order SG, imposing both three-fold rotation and reflection symmetries reduces the number N of equations and variables $\{\gamma_{ij}\}$ to $N=(3^n+1)/2$. This system of equations can be solved to obtain the ground state energy E(f) of a single n^{th} order SG. The result is shown in Fig.2a.

To calculate L_k^{-1}, we characterize each junction by its equivalent inductance $L_{ij} \propto 1/\cos\gamma_{ij}$, where the $\{\gamma_{ij}\}$ are those of the ground states computed above. Then, using Kirchhoff's laws as in electrical networks, we obtain $L_k^{-1}(f)$ of a single SG as shown in Fig.2b.

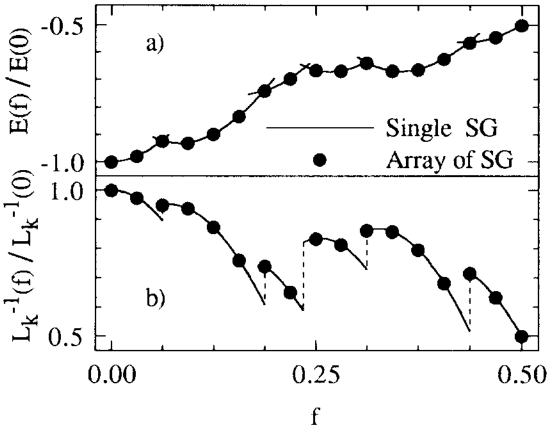

Fig.2: (a) Ground state energy E and (b) corresponding L_k^{-1} for a single 2^{nd} order SG and for an array of 2^{nd} order SG as a function of f.

Let's now come to the case of the periodic repetition of SG. Connecting the n^{th} order SG together creates triangular loops of linear size $2^n \times a$. Expressing flux quantization in these triangular loops introduces an additional equation. As a consequence, the system of N+1 equations becomes overdetermined for every value of f, except for multiples of $f_n=1/(2\times 4^n)$, where the additional equation is trivially satisfied. The multiples of f_n are exactly the values emerging from the nesting property proven in Ref. [6]. Translated into our context, this property allows to assemble three n^{th} order SG together in one $(n+1)^{th}$ order SG without changing the energy (or L_k^{-1}). It follows that, for the discrete set of multiples of f_n, periodic boundary conditions are readily satisfied and, accordingly, the energy (or L_k^{-1}) of the 2D array is determined by that of a single n^{th} order SG as illustrated in Fig.2. For all other frustrations, the ground state of the periodic array cannot be found from a single gasket calculation since it will have a larger elementary cell.

4. Discussion and conclusions

In Fig.1, the measured oscillations are exactly located at multiples of f_n (with n=2) showing that the array is in a phase coherent state for these values of f only. Nonetheless, the amplitudes of many peaks are depressed with respect to our T=0 calculations, showing that thermal fluctuations are important. It seems also natural that no peaks occur at fractional multiples of f_n, since they would correspond to ground states with larger elementary cells and, therefore, are more vulnerable to fluctuations. In contrast to the case of a genuine periodic array [7] in which a non-monotonic sequence of peaks results from the formation of superlattice states, in the present experimental situation, the peaks of different heights correspond to states with the same size of the unit cell and are therefore equally spaced along the f axis.

Finally we notice that, if f_n falls below experimental resolution, the discrete nature of $L_k^{-1}(f)$ (Fig.1) will be washed out. This situation is met in periodic arrays of higher-order SG, whose response was found to be similar to that of a single SG [4].

This work was supported by the Swiss National Science Foundation.

References

[1] Y. Gefen, A. Aharony, B. B. Mandelbrot and S. Kirkpatrick, Phys. Rev. Lett. **47**, 1771, (1981).

[2] J.M.Gordon, A.M.Goldman and B.Whitehead, Phys. Rev. Lett. **59**, 2311 (1987).

[3] B.Jeanneret, J.L.Gavilano, G.A.Racine, Ch. Leemann and P.Martinoli, Appl. Phys. Lett. **55**, 2336 (1989)

[4] R.Meyer, J.L.Gavilano, B.Jeanneret, R.Théron, Ch.Leemann, H.Beck and P.Martinoli, Phys. Rev. Lett. **67**, 3022 (1991).

[5] A.Ceccatto, S.Doniach, K.Frahm and B.Mühlschlegel, Z. Phys. B **82**, 257 (1991).

[6] R.Rammal and G.Toulouse, Phys. Rev. Lett. **49**, 1194 (1982).

[7] R. Théron, J.B. Simond, J.L. Gavilano, Ch. Leemann and P. Martinoli, Physica B **165&166**, 1641 (1990).