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Absence of Reentrance in the Two-Dimensional XY-Model with Random Phase Shift

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Abstract. — We show that the 2D XY-model with random phase shifts exhibits for low temperature and small disorder a phase with quasi-long-range order, and that the transition to the disordered phase is *not* reentrant. These results are obtained by heuristic arguments, an analytical renormalization group calculation, and a numerical Migdal-Kadanoff renormalization group treatment. Previous predictions of reentrance are found to fail due to an overestimation of the vortex pair density as a consequence of the independent dipole approximation. At positions where vortex pairs are energetically favored by disorder, their statistics becomes effectively fermionic. The results may have implications for a large number of related models.

We reconsider in this paper the 2-dimensional XY-model

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j - A_{ij}) \quad (1)$$

with quenched random phase shifts A_{ij} on the bonds, where i, j run over the sites of a square lattice. For simplicity we assume that the A_{ij} on different bonds are uncorrelated and Gaussian-distributed with mean zero and variance σ .

Model (1) describes for example 2-dimensional XY-magnets with random Dzyaloshinskii-Moriya interaction [1]. Other realizations are given by Josephson-junction arrays with positional disorder [2] and model vortex glasses [3]. In particular, in the case of the so-called gauge glass model, one assumes A_{ij} to be uniformly distributed between 0 and 2π . We expect, that our model with Gaussian disorder is equivalent to the gauge glass model when $\sigma \rightarrow \infty$.

For vanishing A_{ij} , model (1) undergoes a Kosterlitz-Thouless (KT) transition, at which the spin-spin correlation exponent η jumps from $1/4$ to zero [4].

Weak disorder, $\sigma \ll 1$, should not change much this picture. In the spin-wave approximation one obtains $\eta = \frac{1}{2\pi}(T/J + \sigma)$, which remains now finite, even at $T = 0$. The features of the KT-transition are essentially preserved, but the transition is shifted to lower temperatures and the jump of η at the transition is diminished [1]. The actual transition temperature $T_c(\sigma) \leq T_+(\sigma)$

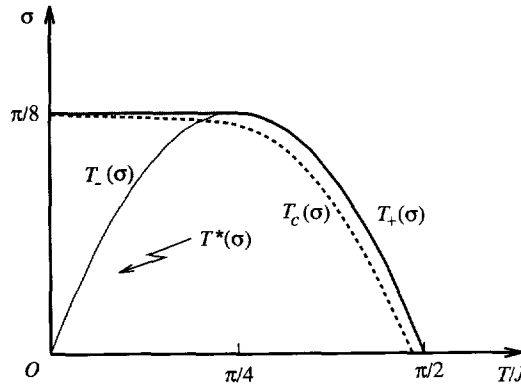


Fig. 1. — (σ, T) phase diagram of the model (1). $T_{\pm}(\sigma)$ are the upper bounds for the transition temperatures $T_c(\sigma)$ and $T_{re}(\sigma)$ between the disordered and the KT phase in the RSN-theory [1]. Note, that T_- -line lies completely in the freezing region (hatched area). The true phase transition line $T_c(\sigma)$ is denoted by the dashed line which is bounded by $T_+(\sigma)$ and $\sigma = \pi/8$. The line T_{re} is not shown here.

jump of η at the transition is diminished [1]. The actual transition temperature $T_c(\sigma) \leq T_+(\sigma)$ depends on the bare value for the vortex core energy E_c , here $T_{\pm} = \frac{\pi}{4} J [1 \pm (1 - 8\sigma/\pi)^{1/2}]$. In the limit $E_c \rightarrow \infty$, $T_c = T_+$.

Strong disorder will suppress the quasi-long-range order of the KT phase [3]. In particular, if $Q = \frac{1}{2\pi} \sum_{\langle \text{plaq} \rangle} A_{ij}$ is of the order one, vortices are generated even at zero temperature. Here

$\sum_{\langle \text{plaq} \rangle}$ denotes the sum over the four bonds of an elementary plaquette.

Rubinstein, Shraiman and Nelson (RSN) [1] extended the Coulomb gas description of the KT-transition [4] to the presence of randomly frozen dipoles arising from the random phase shifts. Surprisingly, they found a second (reentrant) transition at $T_{re}(\sigma) (\leq T_-(\sigma))$ to a disordered phase at low temperatures (see Fig. 1). $T_{re}(\sigma)$ bends towards higher temperatures for increasing disorder. The two lines T_{\pm} merge at $\sigma_c = \pi/8$. For $\sigma > \sigma_c$ there is no ordered phase. The precise value of $T_{re}(\sigma)$ depends again on E_c . Similar results were obtained in reference [2].

Korshunov [5] has argued, that the intermediate phase in the range $T_{re}(\sigma) < T < T_c(\sigma)$ with quasi-long-range order is probably *not stable*, if in addition to the screening of Coulomb charges by neutral pairs of charges, considered in [1], screening by larger complexes of charges in different replicas are taken into account.

Experiments [6], as well as Monte Carlo studies [7], indicate no reentrance. Also, Ozeki and Nishimori [8] have shown for a general class of random spin systems, which include (1), that the phase boundary between the KT- and the paramagnetic phase is parallel to the temperature axis for *low T*. Thus they exclude a reentrant transition, provided the intermediate KT phase exists. However, they cannot rule out the possibility that the KT-phase *disappears* completely, as suggested in [5].

We will argue below that the reentrant transition is indeed an artefact of the calculation scheme used in references [1,2] and that the KT-phase is stable at low temperatures with $T_c(\sigma) \rightarrow 0$ for $\sigma \rightarrow \pi/8$ (see Fig. 1). Since the renormalization group (RG) flow equations (4) (see below) derived in [1], which give rise to the reentrant behavior, appear as a subset of the

more general RG equations for XY-systems with additional symmetry-breaking [9] or random fields [10], as well as for solid films with quenched random impurities [11], these systems have to be reconsidered, which we will postpone to forthcoming publications.

For the further discussion it is useful to decompose the Hamiltonian (1) into a spin-wave part \mathcal{H}_{sw} and a vortex part \mathcal{H}_v . Since the phase transition is governed by \mathcal{H}_v , we will omit \mathcal{H}_{sw} completely. In the continuum description, the vortex part can be rewritten in the form (1) (for simplicity, we set the lattice constant equal to unity)

$$\mathcal{H}_v = -J\pi \sum_i m_i \left\{ \sum_{j \neq i} m_j \ln |\mathbf{r}_i - \mathbf{r}_j| + 2 \int d^2r Q(\mathbf{r}) \ln |\mathbf{r} - \mathbf{r}_i| - \frac{E_c}{J\pi} m_i \right\}. \quad (2)$$

The integer vortex charges m_i satisfy $\sum_i m_i = 0$. $Q(\mathbf{r})$ is a quenched random charge field, which is related to the phase shift $\mathbf{A}(\mathbf{r})$ by $2\pi Q(\mathbf{r}) = -\partial_x A_y + \partial_y A_x$. Here we made the replacement $A_{i,j} \rightarrow \mathbf{A}(\mathbf{r})$ by going over to the continuum description. Since

$$[\mathbf{A}]_d = 0, \quad [A_\alpha(\mathbf{r})A_\beta(\mathbf{r}')]_d = \sigma \delta_{\alpha\beta} (\mathbf{r} - \mathbf{r}'), \quad (3)$$

where $[\dots]_d$ denotes the disorder average, the random charges are *anticorrelated*.

The main result of the work of RSN [1] are the RG-flow equations (4a-c) (see also [2,9,10]), which describe the change of J, σ and the vortex number density y after eliminating vortex degrees of freedom up to a length scale e^l

$$\frac{dJ}{dl} = -4\pi^3 \frac{J^2}{T} y^2 \quad (4a)$$

$$\frac{dy}{dl} = \left(2 - \pi \frac{J}{T} + \pi \frac{J^2}{T^2} \sigma \right) y \quad (4b)$$

$$\frac{d\sigma}{dl} = 0. \quad (4c)$$

Here we use the convention that only the exchange constant J is renormalized and the temperature plays merely the role of an unrenormalized parameter. For $\sigma \equiv 0$ equations (4a,b) behave in a well-defined fashion for $T \rightarrow 0$. This becomes clearer if we rewrite the vortex fugacity as $y = e^{-F_c/T}$, where F_c is the (core) free energy of a single vortex on the scale e^l . Then equation (4b) takes the form $dF_c/dl = (\pi J - 2T - \pi \frac{J^2}{T} \sigma) = 2(T_+ - T)(T - T_-)/T$.

For $\sigma > 0$, the last term on the r.h.s. of equation (4b) blows up at low T , leading to the reentrance transition mentioned above. Whereas for high temperatures the $1/T$ coefficient of the σ term is plausible, since thermal fluctuations wipe out the random potential, we do not see a reason that this effect could lead to an *unlimited* growth of the effective disorder strength at very low temperatures. Clearly, equation (4b) cannot be valid at zero temperature. Contrary to RSN [1], we argue, that equations (4a,b) are valid only for sufficiently high temperatures $T \geq T^*(\sigma) > T_-(\sigma)$.

An indication for T^* follows from the flow of the vortex entropy $S_c = -\partial F_c/\partial T$, $\partial S_c/\partial l = 2 - \pi \frac{J^2}{T^2} \sigma + \pi(-\frac{\partial J}{\partial T})(1 - 2\sigma \frac{J}{T})$. Since $\partial J/\partial T \leq 0$, the entropy is *reduced* for $T < T^* = 2J\sigma$, $\sigma \leq \pi/8$, if one goes over to larger length scales. This leads finally to a negative entropy, which we consider as an artefact of the calculation [1] (see also [2,9-11]). values for $T < 2J\sigma$. The vanishing of the entropy in disordered systems usually signals a freezing of the system by approaching T^* from high temperatures [12]. Similarly, the flow of the vortex energy $E_c = F_c + TS_c$, $\partial E_c/\partial l = (1 - 2\sigma \frac{J}{T})\pi(J - T \frac{\partial J}{\partial T})$ leads for $T < T^*$ eventually to negative values of the core energy. Inevitably, this favours multiple occupancy of vortex positions. However,

the resulting vortices of higher vorticity $|m| > 1$ appear even in the presence of disorder much less likely than those with $|m| = 1$: since their energy cost scales as m^2 , whereas their energy gain scales only as m . This effective repulsion of vortices leads for $T < T^*$ to a much smaller vortex density than in the RSN-theory [1], which neglects completely the interaction between vortex dipoles.

For $T < T^*$ we expect the physics to be different from that described by equations (4). Since $T^*(\sigma)$ intersects the RNS phase boundary at $\sigma = \pi/8$ where $T_+ = T_- = J\pi/4$, the whole (T, σ) -range in which reentrance was observed belongs to the freezing region, which has to be reconsidered.

To find the correct behaviour at low temperatures, we consider first the system at $T = 0$. A simple estimate shows, that then vortices will not be relevant if the disorder is weak. Indeed, the elastic energy of an isolated vortex of charge $\pm m$ in a system of radius R is $m^2\pi J \ln R$, which has to be compared with the possible energy gain E_r from the interaction of the vortex with the disorder. If we rewrite the second term in equation (2) as $\sum_i m_i V(\mathbf{r}_i)$, we find $[V^2(\mathbf{r}_i)]_d \simeq 2\pi J^2 \sigma \ln R$. Hence the typical energy gain is $-J(m^2 2\pi\sigma \ln R)^{1/2}$.

In order to find the maximal energy gain, we have to estimate the number $n(R)$ of vortex positions \mathbf{r}_i in which the energies $V(\mathbf{r}_i)$ are essentially uncorrelated. Two vortex positions $\mathbf{r}_i, \mathbf{r}_j$ have independent energies if $[V^2(\mathbf{r}_i)]_d \gg [V(\mathbf{r}_i)V(\mathbf{r}_j)]_d$, a condition which can be rewritten with

$$[(V(\mathbf{r}_i) - V(\mathbf{r}_j))^2]_d = 4\pi\sigma J^2 \ln |\mathbf{r}_i - \mathbf{r}_j| \equiv \Delta^2(\mathbf{r}_i - \mathbf{r}_j) \quad (5)$$

as $\ln |\mathbf{r}_i - \mathbf{r}_j| \approx (1 - \epsilon) \ln R$ with $\epsilon \ll 1$. Thus $n(R) \approx R^{2\epsilon}$ and the maximal energy gain from exploiting the tail of the Gaussian distribution for $V(\mathbf{r}_i)$ is $E_r \approx -2J(m^2\pi\epsilon\sigma)^{1/2} \ln R$. The total vortex free energy at $T = 0$ is therefore

$$F_c \approx J\pi(m^2 - 2(m^2\epsilon\sigma/\pi)^{1/2}) \ln R \quad (6)$$

and hence vortices should be irrelevant for weak disorder $\sigma \ll 1$.

In studying the behavior for $T = 0$ but larger σ we have to take into account the screening of the vortex and quenched random charges by other vortex pairs. This can be done most easily by using the dielectric formalism. Here we follow the treatment of Halperin [13] who showed that screening by vortex pairs with separation between R and $R + dR$ changes the coupling constant $J(R)$ (which corresponds to the inverse dielectric constant) as

$$J(R + dR) = J(R) - 4\pi^2 \sum_{m>0} \alpha_m(R) J^2(R) 2\pi R p_m(R) dR. \quad (7)$$

Here $\alpha_m(R)$ is the polarizability and $p_m(R)$ is the probability density of a pair with charge $+m$ at \mathbf{r}_1 and charge $-m$ at \mathbf{r}_2 , $R = |\mathbf{r}_1 - \mathbf{r}_2|$.

For the calculation of $p_m(R)$ and $\alpha_m(R)$ we use the fact, that the interaction energy between a vortex pair and the disorder is Gaussian-distributed with a width $\Delta(R)$. If the density of pairs is sufficiently small, we may neglect the interaction between pairs and write

$$p_m(R) = \int_{-\infty}^{-2|m|(\pi J \ln R + E_c)} \frac{dV}{\sqrt{2\pi}\Delta(R)} e^{-\frac{V^2}{2\Delta^2(R)}} \approx \sqrt{\frac{\sigma}{2\pi^2 m^2 \ln R}} R^{-\frac{m^2\pi}{2\sigma}}, \quad (8)$$

where the r.h.s. of equation (8) is valid only for $\sigma, E_c/J \ll \ln R$. Since $p_{|m|>1}(R) \ll p_1(R) =: p(R)$, we will neglect double occupancy of vortex positions.

The polarizability $\alpha(R) = \alpha_1(R)$ can be calculated in a similar way and is found to be $\alpha(R) \approx R^2/T^*$ at large R . With $y^2 = R^4 p(R)$ and $l = \ln R$ we get from equations (7) and (8)

for $T = 0$

$$\frac{dJ}{dl} = -4\pi^3 \frac{J}{\sigma} y^2 \quad (9a)$$

$$\frac{dy}{dl} = \left(2 - \frac{\pi}{4\sigma}\right)y \quad (9b)$$

$$\frac{d\sigma}{dl} = 0, \quad (9c)$$

where we again neglected terms of the order σ/l . These are the flow equations, which replace equations (4) at zero temperature. Within this approximation, the system undergoes a phase transition at $\sigma_c = \pi/8$ from a KT to a disordered phase, which is in qualitative agreement with our estimate (6). At σ_c the exponent η shows a universal jump from $1/16$ to zero. For $\sigma > \sigma_c$, y reaches a value of order magnitude unity on the scale $R \approx \xi$ with

$$\xi \propto e^{1/b(1-\pi/8\sigma)}. \quad (10)$$

b is a constant, which depends on the details of the system. For $R > \xi$ our flow equations are no longer valid, since y is no longer small. We identify ξ with the correlation length in the disordered phase.

We now discuss the properties of the system at low but finite temperatures. The T -correction to our free energy estimate (6) are of the order $-2T \ln R$ (or smaller) and hence will not allow a reentrance transition. A more efficient way for thermal fluctuations to influence the low- T behavior would be the generation of uncorrelated frozen charges $Q(\mathbf{r})$. However unlike to random field systems, where uncorrelated random fields are indeed generated from anticorrelated random fields [14], which destroy the ordered phase in 2 dimensions at all non-zero T , we do not see such a mechanism here. The main difference consists in the existence of a double degenerated ground state in the random field system at $T = 0$.

The physics at finite temperature can also be captured within the dielectric formalism. Neglecting again the interaction between vortex pairs at different positions, we calculate the normalized probability for a pair with charges $\pm m$ as

$$p_m(R) = p_{-m}(R) = \left[\frac{e^{-E_m(R)/T}}{\sum_m e^{-E_m(R)/T}} \right]_d, \quad (11)$$

where $E_m(R) = 2m^2(E_c + \pi J \ln R) + m(V(\mathbf{r}_1) - V(\mathbf{r}_2))$ denotes the pair energy. At large R holds $p_{|m|>1}(R) \approx 0$, since the elastic energy cost $\propto m^2$ will be compensated with decreasing probability by an energy gain $\propto m$ due to disorder. We therefore drop occupancies $|m| > 1$. Furthermore, for a given configuration of disorder, one of the two energies $E_{\pm 1}(R)$ is always so large, that the corresponding weight factor $e^{E_{\pm 1}(R)/T}$ can be neglected. After this approximation, the probability for a single pair reads

$$p(R) = p_1(R) = \left[\frac{1}{1 + e^{E_1(R)/T}} \right]_d. \quad (12)$$

Equation (11) thus effectively reduces to the disorder average of the Fermi distribution function. In other words: vortex pairs of vorticity one can be treated as non-interacting fermions. In the limit $T = 0$, where this distribution function becomes step-like, equation (12) immediately reduces to the previous expression (Eq.(8)). At finite temperature, the disorder average in

equation (12) is performed by splitting the integral over the disorder distribution into two contributions corresponding to $E_1(R) \geq 0$. To leading order in R , we find $p(R) \sim R^{-\pi/2\sigma}$ for $0 \leq T \leq T^* = 2J\sigma$, whereas $p(R) \sim R^{-2\pi J/T(1-\sigma J/T)}$ for $T \geq T^*$. Plugging these results into the definition of y , we obtain flow equation (4b) in the whole range $T \geq T^*$, whereas equation (9b) is valid in the whole range $0 \leq T \leq T^*$. Both equations coincide at the boundary $T = T^*$.

We add a few remarks: as long as $E_1(R) \gg T$, the Fermi distribution can be replaced by the Boltzmann distribution, as is usually done in the treatment of the KT transition [4]. The disorder average of the latter yields $p(R) \sim R^{-2\pi J/T(1-\sigma J/T)}$ and hence equation (4b) for all temperatures. However, for $T < T^*$ the condition $E_1(R) \gg T$ is no longer fulfilled for most of the vortex positions (see also our remarks below Eqs. (4)) and hence this approximation breaks down. Indeed, use of the Boltzmann distribution at low temperatures would lead to $p(R) \gg 1$, and the interaction between vortex pairs could no longer be neglected. It is therefore important to calculate $p(R)$ from equation (12). An attempt to improve upon the Boltzmann-approximation consists in expanding equation (12) into a power series in $e^{-E_1(R)/T}$. The n^{th} order term in the expansion yields a contribution $R^{-2\pi J/T(n-n^2\sigma J/T)}$ to $p(R)$. The series is divergent, i.e., for large R , higher order terms are more important than lower order terms, irrespective of temperature. These higher order terms generate contributions to the flow equation dy/dl , which tend to blow up y even faster. One might hence expect an instability of the ordered phase, similarly to the observation of Korshunov [5]. In fact, the above expansion and in particular the replacement of the Fermi- by the Boltzmann-distribution are disqualified *a posteriori*.

We conclude that $dy(l)/dl < 0$ for all $T < T_+$. The polarizability at finite temperatures is given by $\alpha = R^2/(T + T^*)$ for $T < T^*$ and by $\alpha = R^2/(2T)$ for $T > T^*$. Thus $dJ/dl < 0$ holds for all $T < T_+$ which is sufficient to guarantee the absence of reentrant phase topology. In the special case of $E_c \rightarrow \infty$ the phase boundary is given by $T_+(\sigma)$ for $T \geq J\pi/4$ and a horizontal line $\sigma_c = \pi/8$ for smaller T , as shown by bold lines in Fig. 1. This is consistent with the prediction of Ozeki and Nishimori [8] about the existence of a horizontal phase boundary. We expect the critical behavior at $T_+(\sigma)$ as discussed in [1] to be unchanged. At finite core energies, the actual transition temperature will be renormalized to $T_c(\sigma) < T_+(\sigma)$. Its value $T_c(0)$ is given by the KT flow equations without disorder and lies only slightly below $T_+(0)$ for large E_c . For small σ , the critical RG trajectory flows completely in the domain of equation (4b), where weak disorder induces weak additional screening. Therefore $T_c(\sigma)$ will smoothly decrease with increasing σ . We expect this function to end up in $T_c(\pi/8) = 0$ monotonously, since flow equations vary monotonously in parameter space.

Our conclusions about the absence of reentrance are confirmed also by a discretized Migdal-Kadanoff renormalization group (MKRG) scheme [15] for model (1), which we consider in the last part of this paper. Our technique has been shown to be similar to that of José *et al.* [16]. Their approach is based on studying Migdal-Kadanoff recursion relations for the Fourier components of the (spatially uniform) potential.

In the discretized scheme [15] instead of allowing ϕ to be a continuous variable, we constrain it to take one of many discrete values which are uniformly distributed between 0 and 2π . Hamiltonian (1) is now defined for values of ϕ restricted to $2\pi k/q$, where $k = 0, 1, 2, \dots, (q-1)$ and q is a number of clock states. We define

$$J_{\nu_j}(q, k) = J \cos(2\pi k/q - A_{\nu_j}). \quad (13)$$

The recursion relations for $J_{\nu_j}(q, k)$ may be found in [15]. For the random 2D system, the numerical procedure is based on creating first a pool of N_p bonds, each decomposed into q components according to equation (13). One then picks N_p random batches of 4 such bonds (the corresponding rescaling factor is equal to 2) from the pool to generate a new pool of the

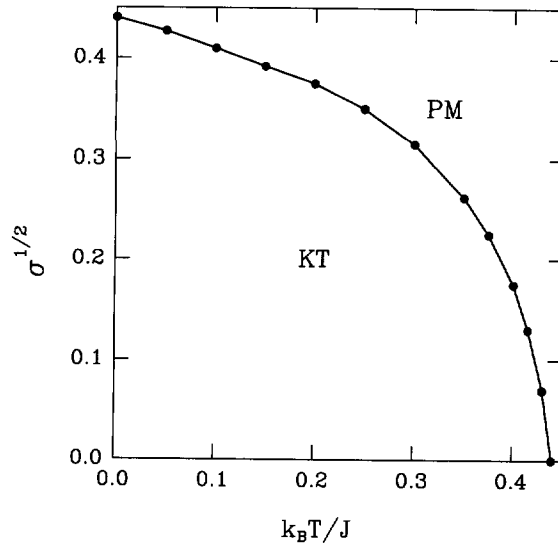


Fig. 2. — (σ, T) phase diagram obtained by the discretized Migdal-Kadanoff RG scheme. PM and KT denote the paramagnetic and Kosterlitz-Thouless phase respectively. In the PM phase $J_{\max}(q, k)$ scales down monotonously, whereas in the KT region it reaches a fixed value at large scales. The critical values $\sigma_c^{1/2}(T=0) \approx k_B T_c(\sigma=0)/J \approx 0.44$. One can also demonstrate that the phase diagram of the random 2D Dzyaloshinskii-Moriya model has the same topology.

coupling variables and the whole procedure is iterated. We consider typically $N_p = 2000$ and $q = 100$. The results depend on these parameters rather weakly.

It should be noted that Gingras and Sorensen [16] have tried to construct the phase diagram of the 2D random Dzyaloshinskii-Moriya model (this model is believed to be equivalent to (1) by the same discretized MKRG approach). In order to locate the Kosterlitz-Thouless phase, they study the scaling behavior of the absolute average height of the potential, \bar{h} , which is defined as follows

$$\bar{h} = \langle |J_{1j}(q, 0) - J_{1j}(q, q/4)| \rangle \quad (14)$$

Due to erratic behavior of \bar{h} they could not draw the phase diagram. The reason here is that \bar{h} representing only two clock states cannot correctly describe the system with many clock states.

To obtain the phase diagram one can consider the scaling of the maximal and minimal couplings for each effective bond or the scaling of the average of absolute values of all q couplings. The scaling properties of these three quantities has been found to be essentially the same, so it is sufficient to focus on the maximum coupling $J_{\max}(q, k)$. The details of this approach can be found in reference [15].

It should be noted that the discretized MKRG approach cannot rigorously reproduce the quasi-long-range XY order in 2D [15]. The scale invariance of $J_{\max}(q, k)$ in the KT-phase is merely approximate in this approach. In practice, the scale invariance of $J_{\max}(q, k)$ persists for about 20 iterations. Further iterations lead to an eventual decrease of $J_{\max}(q, k)$ at any non-zero T . Having this caveat in mind, we can locate the boundary between the paramagnetic and KT-phase (see Fig. 2). Thus the MKRG gives us additional evidence that the reentrance is absent in model (1).

To conclude, in the present paper we have shown by a combination of simple analytical arguments, a renormalization group calculation and a Migdal-Kadanoff RG scheme at finite

T , that the 2-dimensional XY-model with random phase shifts does not exhibit a reentrant transition.

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