Dynamic Measurement of Percolative Critical Exponents in Disordered Josephson Junction Arrays

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The complex conductance $G(\omega)$ of site-diluted Josephson junction arrays close to the percolation threshold was measured over a wide range of frequencies ω. Well below T_c both the superfluid $[\omega \mathrm{Im} G]$ and dissipative $[\mathrm{Re} G]$ components are independent of ω below a critical frequency ω_c, whereas $G(\omega) \propto \omega^{-u}$ with $u \approx \frac{1}{2}$ for $\omega > \omega_c$. This is shown to reflect the crossover from a Euclidean regime ($\omega < \omega_c$) dominated by phononlike modes of the phase system to a fractal regime ($\omega > \omega_c$), where the relevant excitations are localized fractons. Percolative critical exponents extracted from the data are consistent with theoretical predictions. [S0031-9007(96)01507-4]

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Percolation is the simplest idea to understand a disordered system. Near the percolation threshold, percolating systems exhibit a natural self-similar structure with geometrical inhomogeneities occurring over a broad range of length scales and are therefore conveniently described in terms of fractal geometry [1]. With regard to superconductivity, fractal and percolation concepts have proven very useful in acquiring insight into the physics of granular superconductors [2–8]. Although a number of investigations have been performed on disordered granular materials, in most cases the structural aspects of their randomness are so poorly known that a detailed comparison with theoretical predictions is almost impossible. With the advent of modern microfabrication techniques, however, it has become possible to investigate model systems, such as Josephson junction arrays and superconducting wire networks, where both the nature and the amount of disorder can be accurately controlled. Early work has focused on the superconducting-to-normal phase boundary of percolating wire networks exposed to a magnetic field [9,10]. More recently, a Berezinskii-Kosterlitz-Thouless (BKT) transition has been shown to persist in randomly diluted Josephson junction arrays in zero field [11], whereas the unusual scaling properties of vortices as well as the effect of field-induced frustration on superconducting phase coherence have been investigated in a deterministic fractal lattice (the Sierpinski gasket) sharing essential geometrical elements with a truly percolating system near threshold [12,13].

Almost no attention has been paid so far to the *dynamics* of the phase degrees of freedom associated with the randomly distributed superconducting islands in a disordered array. In this Letter we report a study, covering five decades in driving angular frequency ω, of the linear complex ac sheet conductance $G(\omega, p, T)$ of site-diluted triangular arrays of proximity-effect coupled Josephson junctions with site occupation probabilities p very close to the percolation threshold p_c. By exploring the response as a function of ω in zero magnetic field (i.e., at zero frustration) and at temperatures where thermally created vortices are irrelevant, we observe, at a critical value ω_c, a remarkable crossover from a low-frequency ($\omega < \omega_c$) regime, where both $\omega \mathrm{Im} G(\omega)$ (the inverse sheet kinetic inductance measuring superconducting phase coherence in the system) and $\mathrm{Re} G(\omega)$ (the component measuring dissipation) are independent of ω, to a high-frequency ($\omega > \omega_c$) behavior, where $G(\omega) \propto \omega^{-u}$ with $u \approx \frac{1}{2}$. Our theoretical interpretation strongly supports the idea that the crossover in response, observed at $\omega = \omega_c$, reflects the profound change in phase dynamics occurring when $l(\omega)$, the frequency-dependent length scale at which we are probing the system in the conductance measurements, becomes of the order of the percolation correlation length ξ_p [1]. For $l(\omega) > \xi_p$ (i.e., for $\omega < \omega_c$) the array is in the two-dimensional (2D) Euclidean (or homogeneous) regime, where the response is dominated by extended "phononlike" modes of the phase system similar to those occurring in an ordered 2D lattice. In contrast, for $l(\omega) < \xi_p$ (i.e., for $\omega > \omega_c$) the array is in the fractal regime, where localized "fractonlike" phase excitations lead to anomalous dynamics [1,14]. A further unusual feature emerging from our experiments is that the (expected) depression of $\omega \mathrm{Im} G(\omega, p, T)$ caused by percolative disorder is accompanied by additional dissipation, as demonstrated by the discovery of a contribution to $\mathrm{Re} G(\omega, p, T)$ which grows stronger and stronger as $p \to p_c$.

Quite generally, in the classical overdamped limit of interest in this study the sheet conductance of a Josephson junction array follows from a two-fluid description of the system in which the superfluid and the normal fluid are associated, respectively, with the sheet kinetic inductance

L and the sheet resistance R of the array:

$$G = (i\omega L)^{-1} + R^{-1}. \quad (1)$$

Let us first consider an unfrustrated regular ($p = 1$) triangular array driven by a small ac current at temperatures well below the BKT transition temperature T_c. Using a resistively shunted junction model, it is straightforward to show that for $T \ll T_c$, i.e., at temperatures where the phase differences $\{\phi_{jk}(t)\}$ across the junctions are small and, consequently, only plane "phase waves" (the "spin waves" of the classical XY model isomorphic to the array) are the relevant excitations of the system, the array is equivalent to a lattice whose bonds consist of the junction inductance $L_J(T) = (\hbar/2e)^2 J^{-1}(T)$, where $J(T)$ is the temperature-dependent Josephson coupling energy, connected in parallel to the junction resistance $R_J(T)$. Then, it is readily shown that $L = L_J/\sqrt{3}$ and $R = R_J/\sqrt{3}$ for a regular triangular array.

The essential features of the dynamic response of arrays with *percolative disorder* are most easily understood in terms of bond percolation. According to the "universality hypothesis," the main conclusions drawn from this description should also be valid for site percolation (the type of disorder actually present in our samples) on *any* 2D lattice. If one assumes that Josephson couplings $\{J_{jk}\}$ only involve nearest-neighbor pairs $\langle jk \rangle$ of superconducting islands, then bond disorder amounts to set $J_{jk} = J$ on a fraction p of the bonds and $J_{jk} = 0$ on the remaining portion $(1 - p)$. The suppression of a bond $\langle jk \rangle$ also affects the corresponding resistance R_{jk}. However, randomness in the $\{R_{jk}\}$ can hardly be expected to be of any relevance in arrays of proximity-effect coupled junctions, the shunting resistance of the junctions being always finite because of the underlying normal-conducting substrate. Thus, at low enough temperatures (where vortex excitations can be ignored), an array of proximity-effect coupled junctions with bond disorder can be modeled by a two-component random network with elements having conductances $G_1 = (1/i\omega L_J) + (1/R_J)$ and $G_2 = 1/R_J$ with, respectively, probabilities p and $(1 - p)$.

To calculate the sheet conductance $G(\omega, p)$ of the system, we focus on the critical region near p_c relevant to our experiments and consider first of all the case $p = p_c$. Since ξ_p diverges at the percolation threshold, the array is in the fractal regime at all frequencies for $p = p_c$ and, consequently, $G(\omega)$ is expected to obey a power law, $G(\omega) \propto \omega^{-u}$, reflecting the dynamic scaling resulting from the self-similar structure of the system. The dynamical critical exponent u follows by noticing that, because of the self-duality of the problem, the conductance can be calculated exactly [15,16] for bond percolation on a square 2D lattice: $G = (G_1 G_2)^{1/2}$. Substituting the expressions for G_1 and G_2, we then obtain in the limit $\omega\tau_J \ll 1$ of interest ($\tau_J = L_J/R_J$ is the phase relaxation time)

$$L^{-1} = c_L L_J^{-1} (\omega\tau_J)^{1-u}, \qquad R = c_R R_J (\omega\tau_J)^{u}, \quad u = 1/2, \quad (2)$$

where c_L and c_R are numerical coefficients of order unity depending on the structural details of the lattice.

Above p_c, ξ_p is finite and Eq. (2) is no longer valid at all frequencies. Below some crossover frequency ω_c, we must recover the 2D homogeneous regime where both L^{-1} and R are expected to be length scale independent, i.e., independent of ω. Using general scaling arguments [1,17], near p_c the conductance can be written as $G = (G_1 G_2)^{1/2} S(z)$, where $S(z)$ is a complex scaling function and z a scaling variable proportional to $(p - p_c)(G_1/G_2)^{1/2t}$ with t the conductivity exponent [1]. At low frequencies, in the 2D Euclidean regime corresponding to $|z| \gg 1$ or, equivalently, to $\omega\tau_J \ll (p - p_c)^{2t}$, $S(z) \propto z^t(1 + \text{const} \times z^{-2t})$ [17]. Then, denoting by $L_0(p)$ and $R_0(p)$ the sheet kinetic inductance and the sheet resistance in the limit $\omega \to 0$, we find

$$L_0^{-1}(p) = c'_L L_J^{-1} (p - p_c)^t, \qquad R_0(p) = c'_R R_J (p - p_c)^t, \quad (3)$$

where c'_L and c'_R are again numerical factors of order unity depending on the lattice structure. Notice that this result is consistent with the loss of superconducting phase coherence ($L_0^{-1} = 0$) and the formation of the infinite superconducting cluster ($R_0 = 0$) at p_c.

Since the 2D-fractal dynamic crossover is expected to occur for $|z| \sim 1$, using Eq. (3) we obtain the following estimate of ω_c:

$$\omega_c \sim (p - p_c)^{2t}/\tau_J \sim R_0(p)/L_0(p). \quad (4)$$

The crossover at ω_c reflects the drastic change in phase dynamics at the transition from the low-frequency ($\omega < \omega_c$) 2D Euclidean regime, characterized by extended phononlike modes of the phase degrees of freedom, to the high-frequency ($\omega > \omega_c$) fractal regime where localized fractonlike modes are the dominant phase excitations. Calculations [18] based on a self-consistent effective medium approximation [19] reproduce, quite remarkably, the correct value of u and lead to a dynamic behavior similar to that described by Eqs. (2)–(4), however, with $t = 1$.

To test these predictions, we have measured, using a sensitive SQUID-operated two-coil mutual inductance technique [13,20] covering a wide range of driving frequencies (0.1 Hz–20 kHz), the sheet conductance of two site-diluted triangular arrays of proximity-effect coupled Pb/Cu/Pb junctions with percolation fractions $p = 0.55$ and $p = 0.51$ close to the threshold $p_c = 0.50$ [1] and normal-state junction resistances $R_N \approx 7m\Omega$ ($p = 0.55$) and $R_N \approx 3m\Omega$ ($p = 0.51$). Their inverse sheet kinetic inductances at 0.5 Hz, a frequency well below ω_c, are shown in Fig. 1(a) as a function of temperature. Because of their 2D nature at 0.5 Hz, both arrays exhibit, as demonstrated by superfluid drops consistent with the 2D universal prediction [11], a BKT transition at a temperature $T_c(p)$ whose features will be discussed in detail elsewhere. To analyze the superfluid depression caused

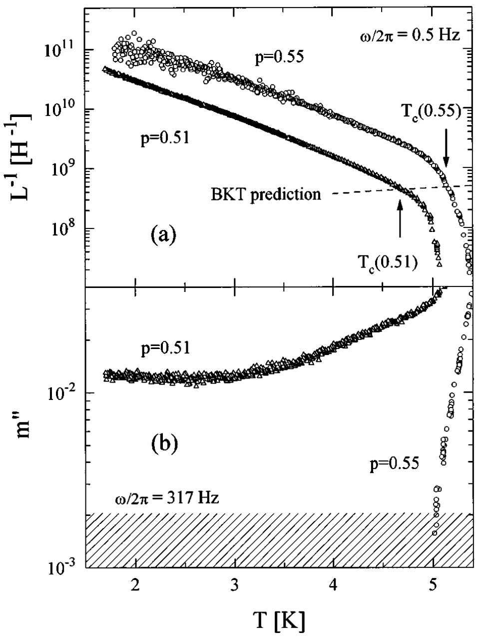

FIG. 1. Temperature dependence of (a) the inverse sheet kinetic inductance at 0.5 Hz and (b) the normalized dissipative component of the mutual inductance change at 317 Hz for two disordered arrays with different percolation fractions on a semilog plot. In (a), the dashed line is the universal prediction for the Berezinskii-Kosterlitz-Thouless transition. In (b), the shaded area is below the sensitivity threshold of the mutual inductance measurements.

by disorder below the critical region, we notice that, below $T_c(p)$, the $L^{-1}(T,p)$ curves manifestly display the same temperature dependence, thereby showing [see Eq. (3)] that the junction inductances $L_J(T) \equiv L_J(0) f(T)$ in both samples differ only in their values $L_J(0)$ at $T = 0$. Then, recalling that $L_J(0) \propto R_N$ [21], the superfluid ratio $L^{-1}(T, 0.55)/L^{-1}(T, 0.51) \approx 4.1$ extracted from Fig. 1(a) can be matched to that given by Eq. (3) by choosing $t \approx 1.4$, in good agreement with the prediction $t \approx 1.3$ for percolation in two dimensions [1].

To illustrate the importance of disorder with regard to dissipation, in Fig. 1(b) we show the temperature dependence of the dissipative component m'' of the mutual inductance change at 317 Hz (still below ω_c) directly detected by the SQUID and caused by the screening currents flowing in the arrays below $T_c(p)$ (m'' is normalized to the purely inductive mutual inductance change at the transition of a perfectly diamagnetic sample). Using a simplified analytical treatment of our measuring technique [20], it can be shown that, well below $T_c(p)$, $m'' \approx C L_J(T)(\omega \tau_J)(R_J/R)^3$ where C is a calibration constant of the order of 10^9 H^{-1}. Since $L_J \approx 0.1$–1 pH and $\tau_J \approx 10^{-7}$–10^{-8} s in the temperature range of interest, at 317 Hz m'' turns out to be ~5 orders of magnitude below our sensitivity threshold ($m'' \approx 0.2\%$, corresponding to an inductance sensitivity of ~1 pH) for a regular array ($R_J/R \approx 1$), whereas for our disordered samples near p_c [$R_J/R_0 \propto (p - p_c)^{-t}$ for $\omega < \omega_c$, see Eq. (3)] m'' should still be below threshold for $p = 0.55$, but well above it (about an order of magnitude) for $p = 0.51$. These predictions are consistent with the low-temperature results shown in Fig. 1(b) which demonstrate the dramatic growth [$\propto (p - p_c)^{-3t}$] of m'' in percolative arrays as $p \to p_c$. We attribute this effect to the scattering of phase (or "spin") waves in a medium whose periodicity is broken by disorder.

The central results of this paper, shown in Figs. 2 and 3, relate to the frequency dependence of G at temperatures well below $T_c(p)$, where our discussion in terms of random networks applies. In Fig. 2 we show, on a log-log plot and at three different temperatures, both $L^{-1}(\omega)$ and $R(\omega)$ over the whole frequency range accessible to our experiments for the array closer to the percolation threshold ($p = 0.51$). The $L^{-1}(\omega)$ data exhibit, at ~1 kHz, a marked crossover from a frequency-independent regime below 1 kHz to a power-law behavior $L^{-1}(\omega) \propto \omega^{(1-u)}$ with $u \approx 0.5$ above 1 kHz. This observation is clearly consistent with the behavior predicted by Eqs. (2) and (3). Although taken, in part, at the limit of our sensitivity and thus lacking the degree of precision achieved in the measurements of the superfluid component, the resistive data $R(\omega)$ also show, at about the same frequency, a crossover consistent with the model predictions, however, with a somewhat larger exponent ($u \approx 0.7$) in the fractal regime. Notice that $R(\omega)$ is strongly temperature dependent, thereby reflecting the reduction of $R_J(T)$ with decreasing temperature caused by the expanding

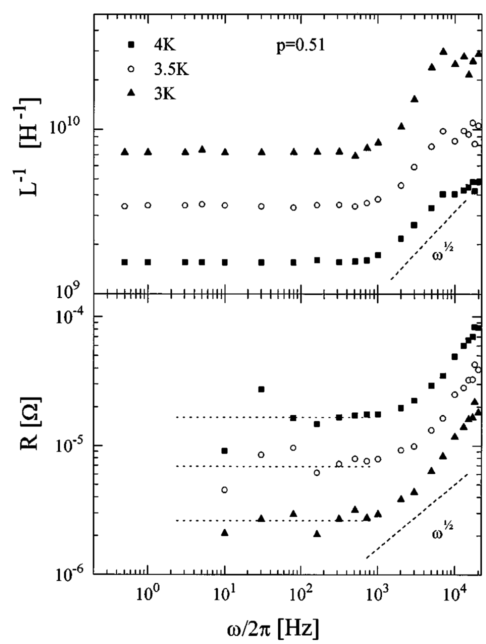

FIG. 2. Frequency dependence of the inverse sheet kinetic inductance L^{-1} and of the sheet resistance R at three different temperatures well below the critical region for the disordered array with $p = 0.51$ on a log-log plot. The dashed lines are $\sqrt{\omega}$ power laws. The dotted lines are guides to the eye to identify the low-frequency plateaus of $R(\omega)$.

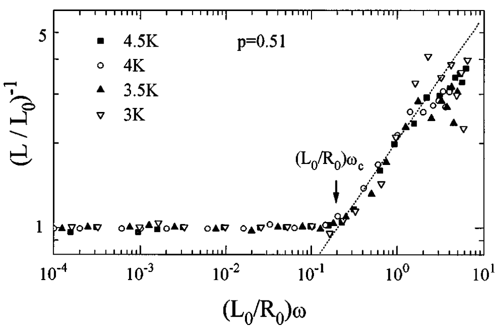

FIG. 3. Normalized inverse sheet kinetic inductance vs normalized angular frequency on a log-log plot showing the universal nature of the phonon-fracton dynamic crossover at $(L_0/R_0)\omega_c$ for the disordered array with $p = 0.51$. The dotted line is a power law with an exponent $1 - u = 0.45$ [Eq. (2)].

superconductivity in the normal Cu link of the Pb/Cu/Pb junctions. Remarkably, the temperature dependence of $R_J(T)$ turns out to be very similar to that of $L_J(T)$, thereby making $\tau_J(T)$ only weakly temperature dependent.

To stress the universal character of the "phonon-fracton" crossover, in the log-log plot of Fig. 3 we show, as a function of the scaling variable $(L_0/R_0)\omega \sim \omega/\omega_c \sim |z|^{-1}$, the normalized inverse sheet kinetic inductance $(L/L_0)^{-1}$ calculated from a collection of data taken at four different temperatures on the sample with $p = 0.51$. $L_0(T)$ and $R_0(T)$ were extracted from the low-frequency plateaus of $L^{-1}(\omega)$ and $R(\omega)$ (see Fig. 2). Within experimental accuracy, all the data collapse on a single curve, thereby demonstrating the scaling of $(L/L_0)^{-1}$ with ω/ω_c predicted by the model [Eqs. (2)–(4)]. From the power-law behavior in the high-frequency fractal regime we deduce $u = 0.55 \pm 0.07$, a value consistent with the theoretical prediction. Moreover, the crossover occurs at $(L_0/R_0)\omega_c \approx 0.2$, a value compatible with the estimate $[(L_0/R_0)\omega_c \approx 1]$ provided by Eq. (4), which entirely neglects numerical factors.

In conclusion, a study of the complex sheet conductance of site-diluted Josephson junction arrays near the percolation threshold has provided novel insight into phase dynamics and dissipative processes in disordered superconductors. In particular, by probing the arrays over a wide range of length scales, we have found strong evidence for a crossover from a low-frequency two-dimensional Euclidean regime, where the response is dominated by extended phononlike modes of the superconducting phase, to a high-frequency fractal regime, where the relevant phase excitations are localized fracton modes. Percolative critical exponents inferred from the analysis of the data are found to be consistent with theoretical predictions.

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